



FACULTY OF ENGINEERING AND TECHNOLOGY

**DEPARTMENT OF ELECTRONICS AND
INSTRUMENTATION ENGINEERING**

VI SEMESTER-B.E.-ELECTRONICS AND INSTRUMENTATION ENGINEERING

CONTROL SYSTEMS LABORATORY

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DEPARTMENT OF ELECTRONICS AND INSTRUMENTATION
ENGINEERING

VI SEMESTER-ELECTRONICS AND INSTRUMENTATION

CONTROL SYSTEMS LABORATORY

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Reg. No. _____ of VI semester B.E Electronics & Instrumentation
Engineering class in Control Systems Laboratory during the year 2013-2014.*

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Department of Electronics and Instrumentation Engineering

B.E. (E & I) VI semester (2013-2014)

CONTROL SYSTEMS LAB

LIST OF EXPERIMENTS

1. Determination of transfer function of a DC servomotor and its speed control
2. Solving control engineering problems using Matlab software
3. Study of DC position control system
4. Design and implementation of a phase lead compensator using Matlab software
5. Identification of a given system using frequency response characteristics
6. (A) Characteristics of sample and hold circuit
(B) Simulation of a sampled data control system
7. (A) Sensitivity analysis of open loop and closed loop systems using process control simulator
(B) Stability characteristics of feedback systems using process control simulator
8. Time response analysis of a second order type-0 and type-1 systems using process control simulator

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CS lab-E&I-FEAT-AU

Exp.No:

Date :

DETERMINATION OF TRANSFER FUNCTION OF A DC SERVOMOTOR AND ITS SPEED CONTROL

(A) TRANSFER FUNCTION OF A DC SERVO MOTOR

AIM

To determine the transfer function of the dc servomotor.

MAJOR APPARATUS REQUIRED

DC Servomotor
Tacho unit
Servo amplifier
Regulated Power Supply (RPS)
Attenuator unit
Large disc
Small disc
Storage CRO
DMM
10K Potentiometer (POT)

THEORY

A DC motor is an electrical machine that converts electrical energy into mechanical energy. These motors are expensive because of brushes and commutators. The DC motors are generally used for large power application as in machine tools and robotics. Except for minor differences in constructional features, DC servomotor is essentially an ordinary DC motor.

A DC servomotor has an output shaft that can be positioned to a specific angular position by sending a command signal and as long as this signal exists the angular position of the shaft is maintained. The servomotor has same control circuits and a potentiometer that is connected to the output shaft. The potentiometer allows the control circuits to monitor the current position of the shaft.

The speed of a DC servomotor can be controlled by two methods namely Armature controlled method and Field control method. The purpose of a motor speed controller is to

take a signal representing the demanded speed and to drive the motor at that speed. Servos are used in radio controlled airplane to position control surfaces like elevators and rudders. They are extremely used in robotics.

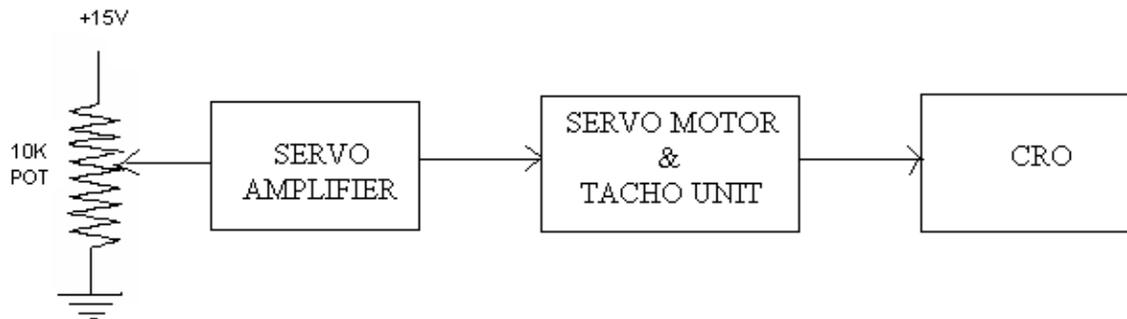


Fig. 1 Connection Diagram to determine the Transfer Function of a DC Servomotor

PROCEDURE

1. Connections are given as per the diagram shown in Fig. 1.
2. Set the motor to run at its rated speed by adjusting the reference input voltage.
3. After the rated speed of the motor is set, do not disturb the settings and switch off the RPS.
4. Keep the Volts/div knob of CRO at 2V and Time/div knob of CRO at 5 secs.
5. Switch on the RPS.
6. When the output waveform reaches the steady state, hold and trace the waveform in CRO and switch off the RPS. Determine the time constant τ , from the waveform. Now measure the input voltage (Volts) applied to the servomotor to run at rated speed.
7. Repeat the above procedure for loading with small as well as large discs. Enter the measured input voltage and calculate the gain K as indicated in Table 1.

8. The transfer function is given by $\frac{K}{1 + \tau s}$ where K is the gain in (deg/secs/volts), and τ in secs.

Table 1 Transfer Function of DC Servomotor

SI. No	LOAD CONDITIONS	INPUT VOLTAGE (volts)	SPEED (deg/sec)	GAIN K (deg/sec/volt)	GAIN K (rad/sec/volt)	TRANSFER FUNCTION
1	Without disc		6000			
2	With small disc		6000			
3	With large disc		6000			

CALCULATIONS

1. For without disc

$$\text{Gain (K)} = \frac{\text{output (deg/sec)}}{\text{input (volts)}} = \frac{\text{speed (deg/sec)}}{\text{input voltage (volts)}} =$$

2. For small disc

$$\text{Gain (K)} = \frac{\text{output (deg/sec)}}{\text{input (volts)}} = \frac{\text{speed (deg/sec)}}{\text{input voltage (volts)}} =$$

3. For large disc

$$\text{Gain (K)} = \frac{\text{output (deg/sec)}}{\text{input (volts)}} = \frac{\text{speed (deg/sec)}}{\text{input voltage (volts)}} =$$

To compute the **Time constant (τ)**, note down the time taken by the output waveform to reach 63.2% of final steady state value.

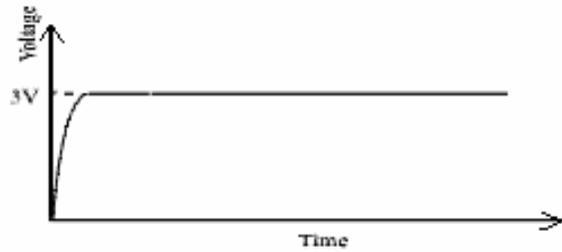


Fig. 2(a) Step response of DC Servomotor under unloaded condition

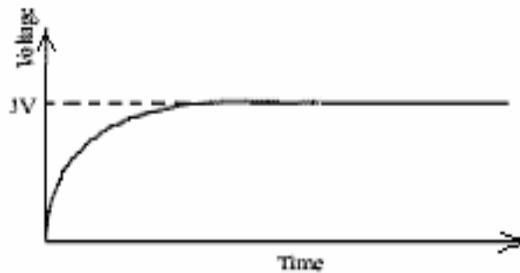


Fig. 2(b) Step response of DC Servomotor under loaded condition (small disc)

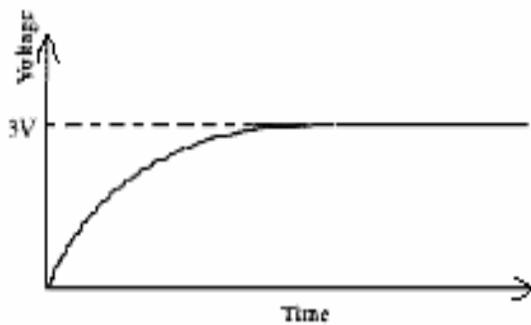


Fig. 2(c) Step response of DC Servomotor under loaded condition (large disc)

CONVERSION (Speed in terms of volts)

Since 3volts correspond to a speed of 1000 rpm

$$\begin{aligned} 1000 \text{ revolutions/min} &= \frac{1000}{60} \text{ rev / sec} \\ &= \frac{1000 * 360}{60} \text{ deg/sec [ie. 1 rev = 360 degree]} \\ &= 6000 \text{ deg/sec} \end{aligned}$$

Hence 3 volts correspond to 6000 deg/sec.

RESULT

The transfer functions of the DC servomotor for different load conditions are determined.

1. Transfer function with no load =

2. Transfer function with small disc =

3. Transfer function with large disc =

(B) SPEED CONTROL OF DC SERVOMOTOR

AIM

To determine the speed versus load characteristics of a DC servomotor with and without tacho feedback.

MAJOR APPARATUS REQUIRED

DC servomotor
Tacho unit
Regulated power supply
Servo amplifier
Attenuator and op-amp units
DMM

PRECAUTION

Make sure that the feedback is negative before giving the feedback connections.

PROCEDURE

Op Amp Offset Null Adjustment

1. Connect +12V,-12V and common terminals from regulated supply to the op-amp unit as indicated in Fig. 3.
2. Put the feedback selector switch of op-amp in 100k mode and ground all the input terminals of op-amp.
3. Measure the output voltage. If the voltage is not zero, adjust it to zero volts using the zero adjustment knob available in the op-amp unit.

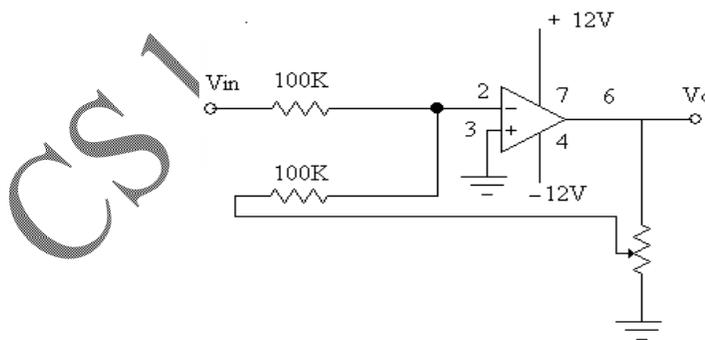


Fig. 3 Amplifier circuit of DC servomotor

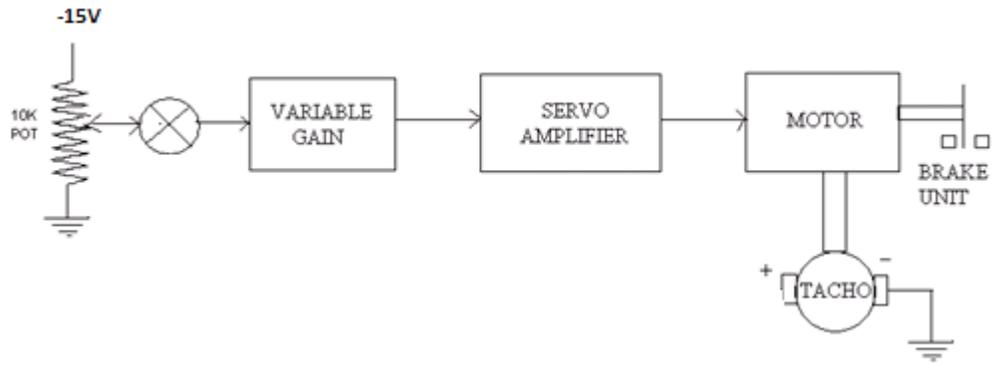


Fig. 4 (a) Connection diagram for DC Servomotor speed control with out tacho feedback

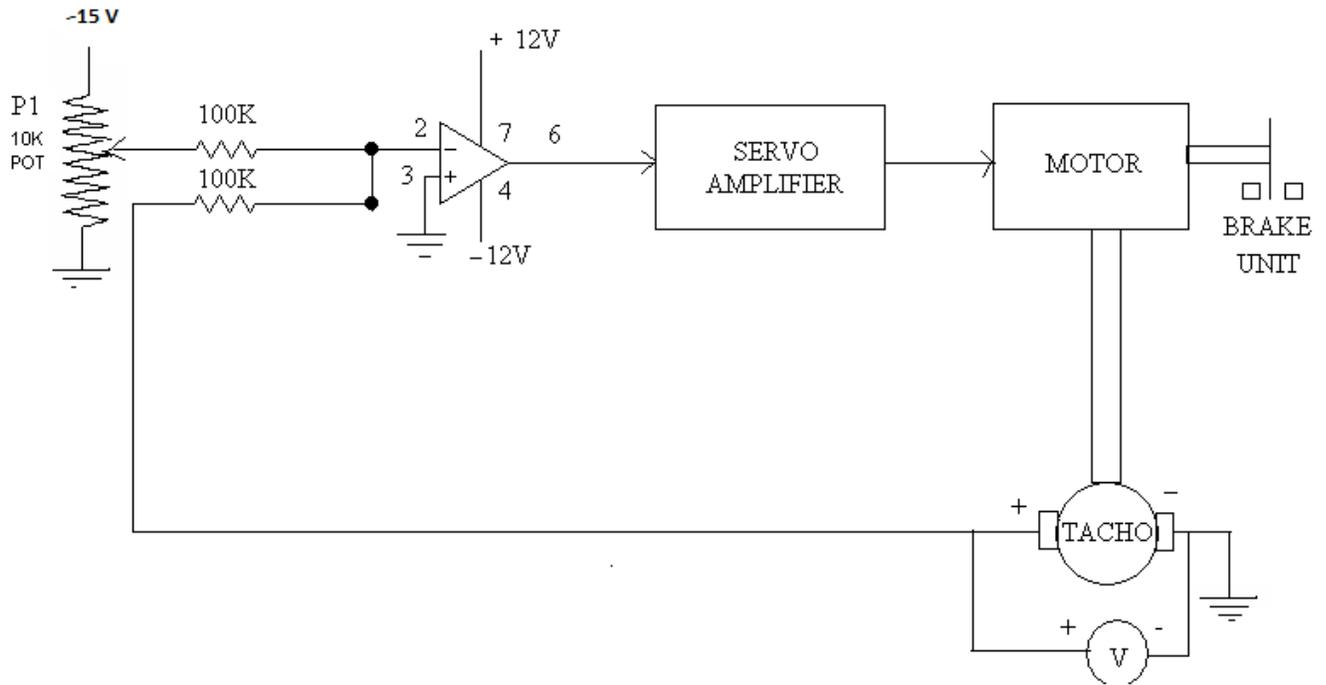


Fig. 4 (b) Connection diagram for DC Servomotor speed control with tacho feedback

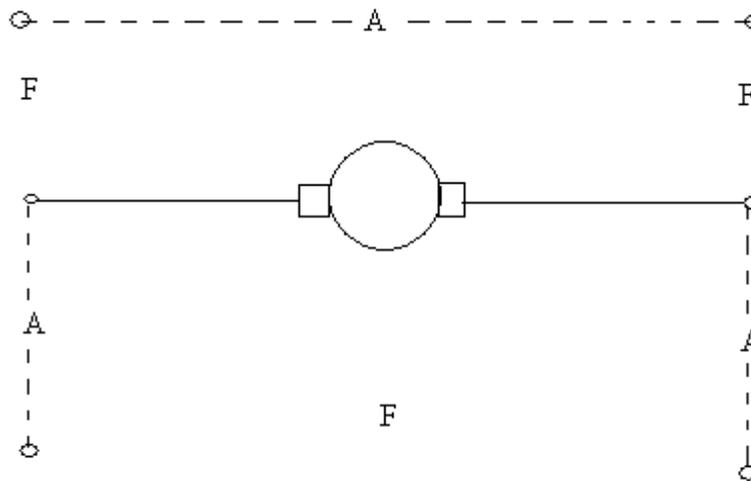


Fig. 5 Armature control connection diagram

WITHOUT TACHO FEEDBACK

1. In order to study the DC servomotor speed control without tacho feedback, make the connections as per the circuit diagram shown in Fig. 4(a).
2. Set the op-amp feedback selector switch to external feedback position.
3. Make the panel connections in servo amplifier for armature control as shown in Fig. 5.
4. Set the pot P2 at 10k (gain $K = 1$), pot P1 in minimum position and load at zero.
5. Switch on the power supply.
6. By varying the pot P1 adjust the speed of the motor to rated speed (1000rpm).
7. This is done by measuring the tachogenerator voltage using DMM (1000 rpm corresponds to 3volts).
8. Now pot P1 should not be disturbed.

9. Apply load through brake magnet and note down the voltmeter reading and enter as in Table 2.
10. Repeat the same procedure for various positions of the brake magnet. Draw a graph between speed and load as shown in Fig. 6.

WITH TACHO FEEDBACK

1. Connections are made as per the circuit diagram shown in Fig. 4(b).
2. To check the negative feedback allows the motor to run by applying a small reference voltage and varying the pot P1 from minimum position.
3. Take the output of the tacho terminals and make contact with the corresponding op-amp unit input terminals.
4. Check whether the speed increases or decreases.
5. If the speed increases the feedback is positive. If the speed decreases the feedback is negative.
6. If the feedback is positive interchange the tacho output terminals.
7. Now set the pot P2 at 10K and the load at zero.
8. By adjusting pot P1 bring the motor to rated speed.
9. Apply the load and note down the output voltage and enter as in Table 3.
10. Repeat the above procedure for various brake magnet positions and with different gains. Draw a graph between speed and load as shown in Fig. 6.

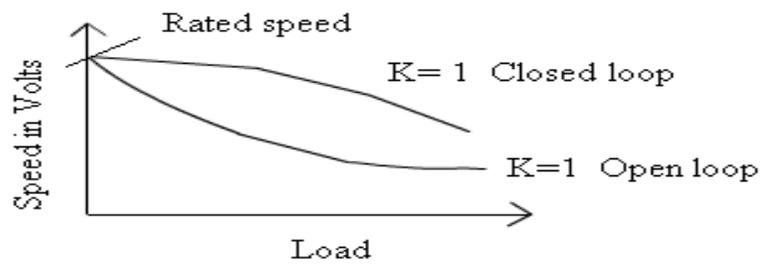


Fig. 6 Speed Vs Load characteristics

Table 2 Load Vs speed characteristics without tacho feedback

Load	Speed in Volts (for Gain $K=1$)

Table 3 Load Vs speed characteristics with tacho feedback

Load	Speed in Volts (for Gain $K=1$)

RESULT

The load versus speed characteristics of DC servomotor with and without tacho feedback are obtained.

CS lab-E&I-FEAT-AU

Exp.No:

Date :

SOLVING CONTROL ENGINEERING PROBLEMS USING MATLAB SOFTWARE

AIM

1. To study the features available in MATLAB software and to obtain the response of a given system subjected to step input through the simulation.
2. To obtain the Root Locus of the given system using MATLAB software.
3. To obtain the Bode plot and Nyquist plots of the given system using MATLAB software.

INTRODUCTION

MATLAB is a high performance interactive software tool for Engineering numeric analysis and scientific computation. Its toolboxes have a collection of “*m*”-files that implement application specific commands.

The *m*-files contain a sequence of ordinary MATLAB statements and it may receive parameters from the command line and return parameters to the workspace. “*m*” in file name of *m*-file refers to the extension used to store the files on the disk.

Tool boxes available in MATLAB relevant to Control Engineering include:

- ❖ Control system toolbox
- ❖ Simulink toolbox
- ❖ System identification toolbox
- ❖ Data acquisition toolbox
- ❖ Robust control toolbox
- ❖ Neural network toolbox

CONTROL SYSTEM TOOLBOX

It provides functions for common control system design, modeling and analysis. Time and Frequency responses, root locus measure and other designs are supported in both continuous and discrete forms.

SIMULINK

It is a program for simulating dynamic system. The user can simulate and analyze dynamic systems from a block diagram description. SIMULINK also supports a number of non-linear blocks including relays, hysteresis and others.

DATA ACQUISITION SYSTEM

The toolbox is a hardware/software combination for acquiring real-time data directly into MATLAB work space.

NEURAL NETWORK TOOLBOX

This is used for designing and simulating artificial neural network. It has functions for different learning rules, training strategies and transfer function.

MATLAB FUNDAMENTALS

MATLAB uses an expression language which is interpreted on the command line. A statement is typically of the form

$$[X_1, X_2, \dots, X_n] = \text{command} [Y_1, Y_2, \dots, Y_n]$$

the left hand side variables are used to store the results of operation command evaluated with right hand side variables as arguments, For example,

To enter a 2x2 matrix:

>> command prompt

>> M = [1 2; 0 4]

M = 1 2

M = 0 4

The matrix is stored in the variable M. Operations such as addition, subtraction, matrix inversion, Eigen value calculations are all built in function of the interpreter.

A powerful feature of MATLAB is its extensive range of graphing capabilities, Utilities exist for linear and logarithmic scales, polar plots, contour plots and bar charts.

EXAMPLE 1 (for Step Response)

The transfer function of a system is given by

$$G(s) = \frac{K}{s + 2}$$

In order to obtain the step response for this system the following statements are to be entered. The coefficients of the polynomial should be entered in descending powers of s Assume K=1.

MATLAB COMMANDS TO OBTAIN STEP RESPONSE

```
num = [0 1];  
den = [1 2];  
t = 0: 0.05: 10;  
figure (1);  
step (num, den, t)
```

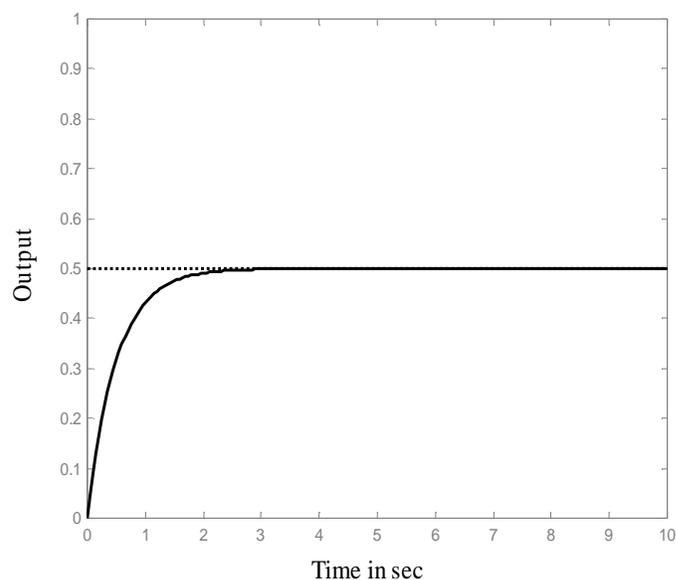


Fig.1 Step response of a System as in Example (1)

The function step (.....) is used to display the step response on the screen. The variable "K" is used to override the automatic range selection of the step command, setting the time range from 0 to 10s in 0.05 s increment. The step response for the above system for K =1 is shown in Fig.1.

FIGURE Create figure window.

FIGURE, by itself, creates a new figure window, and returns its handle. FIGURE (H) makes H in the current figure, forces it to become visible, and raises it above all other figures on the screen. If figure H does not exist, and H is an integer, a new figure is created with handle H.

ROOT LOCUS Plots Root locus.

RLOCUS (SYS) computes and plots the root locus of the single-input, single-output LTI model SYS. The root locus plot is used to analyze the negative feedback loop and shows the trajectories of the closed loop poles when the gain 'K' varies from zero to infinite. RLOCUS automatically generates a set of positive gain values that produce a smooth plot. RLOCUS (SYS,K) uses a user-specified vector K of gain values.

[R,K]=RLOCUS (SYS) or R= RLOCUS (SYS,K) returns the matrix R of complex root locations for the gains K. R has LENGTH (K) columns and its j-th column lists the closed-loop roots for the gain K(j).

CONV: Convolution and Polynomial multiplication.

C=CONV(A,B) convolves vectors A and B. the resulting vector is length LENGTH (A) + LENGTH(B) -1. If A and B are vectors of polynomial coefficients, convolving them equivalent to multiplying the two polynomials.

EXAMPLE 2

Consider the transfer function of a system

$$G(s) = \frac{1}{s(s+1)(s+5)}$$

MATLAB COMMANDS TO OBTAIN ROOT LOCUS PLOT

```
num = [0 1];  
den1 = [1 0];  
den2 = [1 1];  
den3 = conv(den1,den2);  
den4 = [1 5];  
den = conv(den3,den4);  
figure(2);  
rlocus(num,den);  
title('ROOT LOCUS')
```

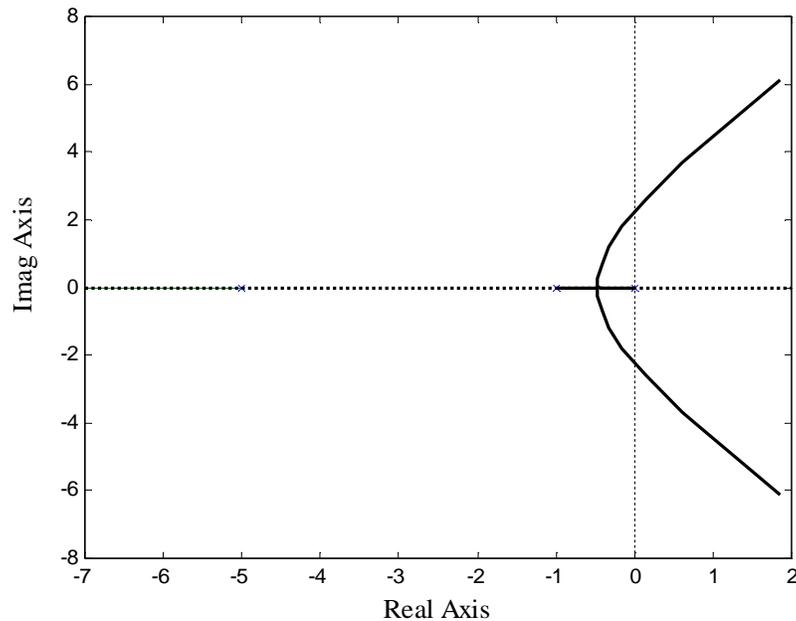


Fig.2 Root Locus Plot of system as in Example (2)

The Root Locus of the system represented by above equation is shown in Fig.2.

BODE Bode frequency response of Linear Time Invariant (LTI) modes.

BODE (SYS) draws the Bode plot of the LTI model SYS (created with TF, ZPK, SS, or FRD). The frequency range and number of points are chosen automatically.

BODE (SYS,[WMIN, WMAX]) draws the Bode plot for frequencies between WMIN and WMAX (in radians/second) with NY outputs and NU inputs.

MARGIN Gain and phase margins and crossover frequencies.

[G_m, P_m, W_{cg}, W_{cp}] = MARGIN (SYS) computes the gain margin G_m, the phase margin P_m in degrees, and the associated frequencies W_{cg}, and W_{cp}, for a SISO open-loop LTI model SYS (continuous or discrete). The gain margin G_m is defined as 1/G where G is the gain at -180° phase crossing. The gain margin in dB is 20 log₁₀(G_m).

[G_m, P_m, W_{cg}, W_{cp}] = MARGIN (MAG, PHASE, W) derives the gain and phase margins from the Bode magnitude, phase, and frequency vectors namely MAG, PHASE, and W produced by BODE command. Interpolation is performed between the frequency points to estimate the values.

For S₁.....S_p array SYS of LTI models, MARGIN returns arrays of size [s₁.....s_p] such that [G_m (j₁,...,j_p), P_m(j₁,...,j_p)] = MARGIN (SYS (:,:,j₁,...,j_p)).

When invoked without left hand arguments, MARGIN (SYS) plots the open loop Bode plot with the gain and phase margins marked with a vertical line.

LOGSPACE Logarithmically spaced vector.

LOGSPACE (X1,X2) generates a row vector of 50 logarithmically equally spaced points between decades 10^{X1} and 10^{X2}, If X2 is pi, then the points are between 10^{X1} and pi. LOGSPACE (X1, X2, N) generate N points

BODE (SYS, W) uses the user-supplied vector W of frequencies in radian/second, at which the Bode response is to be evaluated.

BODE (SYS1, SYS2...W) plots the Bode response of multiple LTI models SYS1, SYS2... on a single plot. The frequency vector W is optional. One can also specify a color, line style, and marker for each system, as in bode (sys1,'r', sys2, 'y', sys3, 'gx').

[MAG, PHASE] = BODE (SYS, W) and [MAG, PHASE, W] = BODE (SYS) return the response magnitudes and phases in degrees (along with the frequency vector W if unspecified). No plot is drawn on the screen. If SYS has NY outputs and NU inputs, MAG and PHASE are arrays of size [NY NU LENGTH (W)] where MAG (:,:,j,...,K) and PHASE

(:,:,.....,K) determine the response at the frequency W(k). To get the magnitudes in dB. type
 $MAGDB = 20 \cdot \log_{10} (MAG)$

For discrete-time models with sample time Ts, BODE uses the transformation $Z = \exp(j \cdot W \cdot Ts)$ to map the unit circle to the real frequency axis. The frequency response is only plotted for frequencies smaller than the Nyquist frequency π/Ts , and the default value (1sec) is assumed when Ts is unspecified.

EXAMPLE 3 (for frequency response)

Consider the transfer function of a system

$$G(s) = \frac{6}{s^3 + 2s^2 + 5s + 2}$$

To draw the Bode Plot (magnitude phase plot) and to determine the gain margin, phase margin, gain crossover frequency and phase cross over frequency. The following commands and statements need to be entered from the command line.

MATLAB COMMANDS TO OBTAIN BODE PLOT

```
n = [0 0 0 6];  
d = [1 2 5 2];  
w = logspace(-1,3);  
[mag,phase,w]=bode (n,d,w);  
[gm,pm,wcg,wcp]=margin(mag,phase,w);  
figure(3);  
Bode(n,d,w);  
Margin(mag,phase,w)  
Title('BODE PLOT')
```

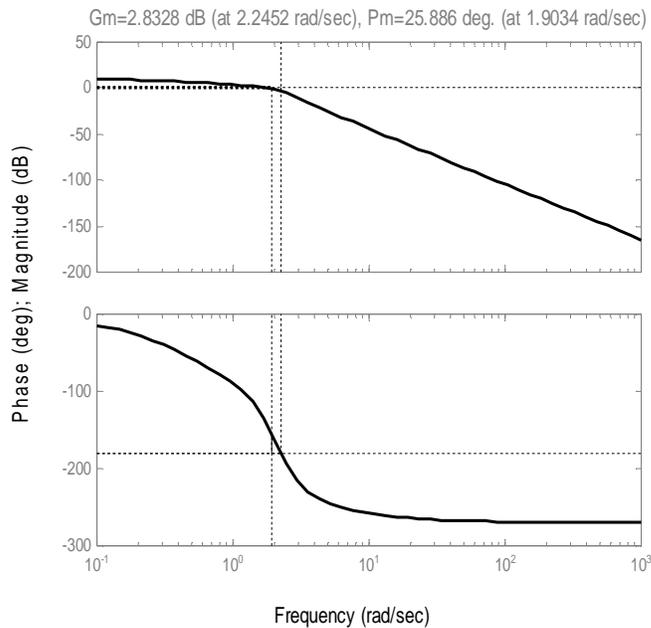


Fig.3 Bode Plot of the System as in Example (3)

The Bode Plot of the system represented by above equation is shown in Fig.3.

NYQUIST Nyquist frequency response of LTI models.

NYQUIST (SYS) draws the Nyquist plot of the LTI model SYS (created with TF, ZPK, SS, or FRD). The frequency range and number of points are chosen automatically.

NYQUIST (SYS, {WMIN, and WMAX}) draws the Nyquist plot for frequencies between WMIN and WMAX (in radians/second).

NYQUIST (SYS, W) uses the user-supplied vector W of frequencies (in radian/ second) at which the Nyquist response is to be evaluated.

NYQUIST (SYS1, SYS2,....., W) plots the Nyquist response of multiple LTI models SYS1, SYS2,....., on a single plot. The frequency vector W is optional. One can also specify a color, line style, and marker for each system, as in

```
nyquist (sys1,'r', sys2, 'y', sys3, 'gx').
```

[RE, IM] = NYQUIST (SYS, W) and [RE, IM, W] = NYQUIST (SYS) return the real parts RE and imaginary parts IM of the frequency response (along with the frequency Vector W if unspecified).

POLAR (THETA, RHO) makes a plot using polar coordinates of the angle THETA, in radians, versus the radius RHO.

POLAR(THETA,RHO,S) uses the line style specified in string S.

MATLAB COMMANDS TO OBTAIN NYQUIST PLOT

```
n=[0 0 0 6];  
d=[1 2 5 2];  
[rp,ip,w]=nyquist(n,d)  
m=[abs(rp+ip*i)];  
a=[angle(rp+ip*i)];  
figure(4);  
polar(a,m);  
[q,p]=margin(n,d)  
title('NYQUIST PLOT')
```

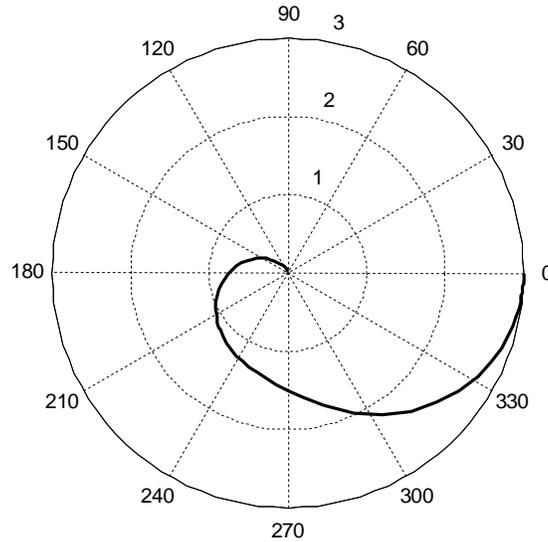


Fig.4 Nyquist Plot of the System as in Example (3)

The Nyquist Plot of the system represented by above equation is shown in Fig.4. The plot crosses the unit circle between -150° and -180° (exactly at -157°). The phase margin is 23° . The plot crosses 180° line before it crosses unit circle is around 0.76. The gain margin is $20 \log (1/ 0.76) = 2.38$ dB.

RESULT

The features and commands available in MATLAB software are studied. The step response of the given system was obtained using MATLAB software.

The Bode plot and the Nyquist plot of the given system are obtained. The Root Locus plot of the given system is obtained. The frequency domain parameters such as PM and GM are determined for both Bode and Nyquist plots and W_{cg} and W_{cp} are determined for bode plot using MATLAB software.

Exp.No:

Date :

STUDY OF DC POSITION CONTROL SYSTEM

AIM

To study the performance characteristics of a DC motor angular position control system.

MAJOR APPARATUS REQUIRED

DC Position Control System kit
CRO
Regulated Power Supply

DESCRIPTION OF DC POSITION CONTROL SYSTEM EQUIPMENT

The schematic diagram of DC motor position control system is shown in Fig.1 and its individual blocks are described as follows.

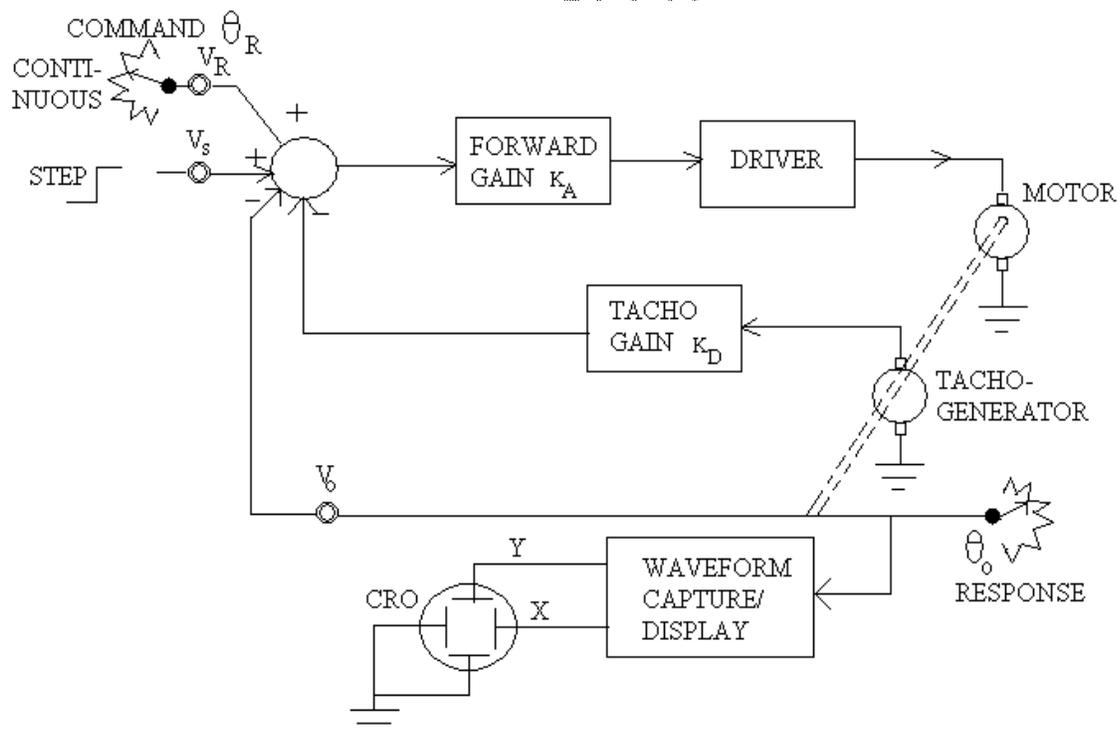


Fig.1 Block Diagram of a DC Motor Position Control System

1. Angle command (continuous): obtained through a 360° servo - potentiometer with a calibrated disc attached.
2. Angle command (step): available through a toggle switch.

2. Motor Unit

The position control is achieved through a good quality permanent magnet DC gear motor. The specifications of the motor are:

1. Operating voltage: 12V d.c.
2. Full load current: 1.2 A.
3. Rated speed: 50 rpm and
4. Torque (basic): 750 gm-cm (approx.)

Angular position of the motor shaft is sensed by a special 360° potentiometer attached to it. A calibrated disk mounted on the potentiometer indicates its angular position in degrees. In addition to this, a small tachogenerator attached to the motor shaft produces a voltage proportional to its speed which is used for velocity feedback.

3. Main Unit

The main unit consists of the following blocks,

Command The continuous command is given by the rotation of a potentiometer through a certain angle. A step command equivalent to about 150 degree may be given by a switch, which is used for quantitative studies of the step response.

Error Detector This is a 4-input 1-output block. Two of the inputs are meant for continuous signals and remaining two inputs, having 180° phase shift, are used for position and velocity feedback signals.

Gain Blocks The forward path gain is adjustable from 0 to 10 and the tachogenerator gain may be varied from 0 to 1 in steps of 0.1.

Driver The driver is unity gain complementary symmetry power amplifier suitable for running the motor up to full power in either direction.

Waveform Capture/Display Unit The waveform capture/display unit is a microprocessor

based card which can 'capture' the motor response and 'display' the same on any ordinary X-Y oscilloscope for a detailed study.

Power Supply The set-up has a number of IC regulated supplies which are permanently connected to all the circuits.

POSITION CONTROL SYSTEM

The position control can be modeled as a second order system

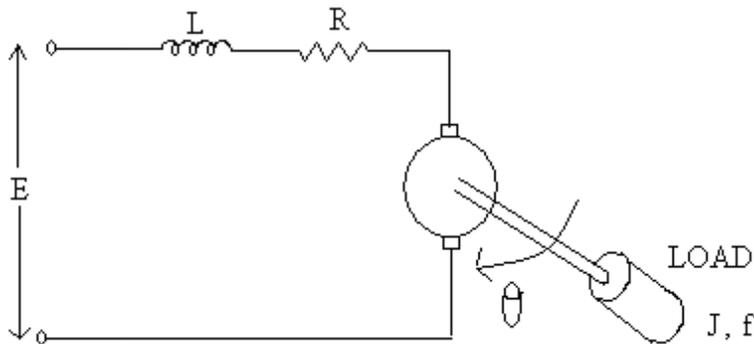


Fig.2 Equivalent Circuit Representation of a DC Motor

A second order system is represented in the standard form as,

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Where δ is called the damping ratio and ω_n the undamped natural frequency. Depending upon the value of δ , the poles of the system may be real, repeated or complex conjugate which is reflected in the nature of its step response. Results obtained for various cases are,

(i) under damped case ($0 < \delta < 1$)

$$c(t) = 1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin\left(\omega_d t + \tan^{-1} \frac{\sqrt{1-\delta^2}}{\delta}\right)$$

where, $\omega_d = \omega_n \sqrt{1-\delta^2}$ is termed the damped natural frequency.

(ii) Critically damped case ($\delta = 1$)

$$c(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$$

(iii) Overdamped case ($\delta > 1$)

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{(\delta^2 - 1)}} \left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right)$$

$$\text{where } s_1 = (\delta + \sqrt{(\delta^2 - 1)})\omega_n \text{ and } s_2 = (\delta - \sqrt{(\delta^2 - 1)})\omega_n$$

Consider the armature-controlled DC motor shown in Fig.2. Let R be the resistance, L be the inductance, I_a be the armature current, I_f be the field current, E be the applied armature voltage, E_b be the back emf, T_M be the torque developed by motor, θ be angular displacement of motor-shaft, J be the equivalent moment of inertia of motor and load referred to motor shaft, f_o be the equivalent viscous friction coefficient of motor and load referred to motor shaft.

The air gap flux Φ is proportional to the field current, i.e.,

$$\Phi = K_f I_f \text{ where } K_f \text{ is a constant.}$$

The torque T_M developed in the armature-controlled DC motor is given by

$$T_M = K_T I_a$$

where K_T is known as the motor torque constant.

The motor back emf being proportional to speed is given as

$$E_b = K_b \frac{d\theta}{dt}$$

where K_b is the back emf constant.

The differential equation is given by

$$L \frac{dI_a}{dt} + RI_a + E_b = E$$

The torque equation is expressed as

$$J \frac{d^2\theta}{dt^2} + f_o \frac{d\theta}{dt} = T_M = K_T I_a$$

Taking laplace transform of above equations assuming zero initial conditions, then the expressions are given by

$$E_b(s) = K_b s \theta(s)$$

$$(L s + R) I_a(s) = E(s) - E_b(s)$$

$$(J s^2 + f_o s) \theta(s) = T_M(s) = K_T I_a(s)$$

From above equations the transfer function of the system is obtained as

$$G(s) = \frac{\theta(s)}{E(s)} = \frac{K_T}{s[(R + sL)(Js + f_o) + K_T K_b]}$$

The armature circuit inductance L is usually negligible. Therefore the transfer function of the armature controlled motor is given by

$$\frac{\theta(s)}{E(s)} = \frac{K_T / R}{Js^2 + s(f_o + K_T K_b / R)}$$

Let $f = f_o + K_T K_b / R$

Then

$$\frac{\theta(s)}{E(s)} = \frac{K_T / R}{s(Js + f)}$$

The transfer function may be written in the form

$$\frac{\theta(s)}{E(s)} = G(s) = \frac{K_m}{s(s\tau + 1)}$$

where $K_m = K_T / R_f =$ Motor Gain Constant and $\tau = J/f =$ Motor Time Constant.

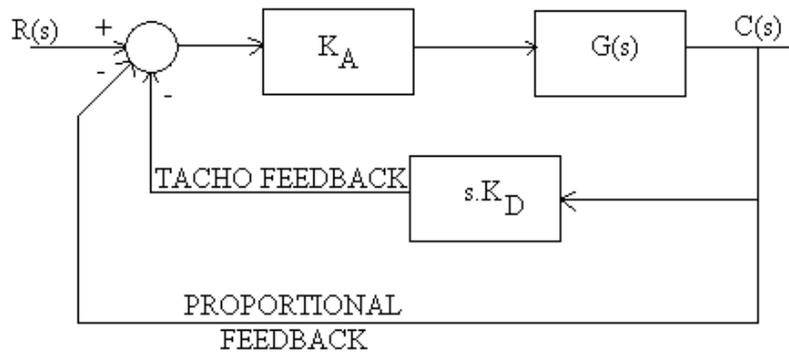


Fig. 3 Simplified Block Diagram of a DC Motor Position Control System

Considering proportional feedback only, the closed loop transfer function of the system of Fig. 3 may be obtained as,

$$\frac{C(s)}{R(s)} = \frac{K_A G(s)}{1 + K_A G(s)} = \frac{K_A K_m / T}{s^2 + s/T + K_A K_m / T}$$

The step response of an underdamped second order system is shown in Fig. 4 and various performance characteristics are listed below.

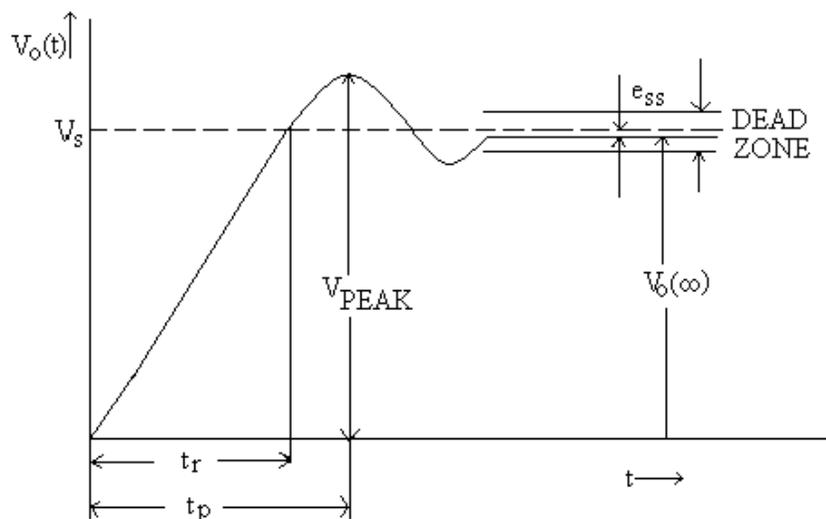


Fig. 4 Typical Step Response of the Position Control System

Delay time (t_d) is defined as the time needed for the response to reach 50% of the final value.

Rise time(t_r) is the time taken for the response to rise from 10% to 90% of the final value for overdamped systems and 0 to 100% of the final value for underdamped systems. This is given by

$$t_r = \frac{\pi - \beta}{\omega_d}, \text{ where } \beta = \tan^{-1} \frac{\sqrt{1 - \delta^2}}{\delta}$$

Peak time (t_p) is the time taken for the response to reach the first peak of the overshoot and is given by

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}}$$

Maximum overshoot (M_p) is defined by

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

Settling time (t_s) is the time required by the system response to reach and stay within a prescribed tolerance band which is usually taken as $\pm 2\%$ or $\pm 5\%$. An approximate calculation based on the envelopes of the response for a low damping ratio system yields

$$t_s (\pm 5\% \text{ tolerance band}) = \frac{3}{\delta \omega_n}$$

$$t_s (\pm 2\% \text{ tolerance band}) = \frac{4}{\delta \omega_n}$$

Steady state error(e_{ss}) is defined as

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{t \rightarrow \infty} \{r(t) - c(t)\}$$

The transient response of the system is affected by the value of K_A . A higher value of K_A should result in larger overshoot.

Tachogenerator Feedback

Considering the tachogenerator feedback path also active in Fig. 3, the closed loop transfer function is obtained as

$$\frac{C(s)}{R(s)} = \frac{K_A G(s)}{1 + K_A G(s)(1 + K_D s)} = \frac{K_A K_m / T}{s^2 + (1 + K_A K_m K_D)(s/T) + (K_A K_m)/T}$$

It is easily seen that the steady state error to unit ramp is given by

$e_{ss} = 1 / K_A K_m$, and the damping ratio is given by

$$\delta = \frac{1 + K_A K_m K_D}{2\sqrt{TK_A K_D}}$$

Thus the specification of e_{ss} and δ may be met simultaneously by a proper choice of K_A and K_D .

PROCEDURE

(a) Position Control through Continuous Command

1. Ensure that the step command **switch is OFF**.
2. Starting from one end move the COMMAND potentiometer in small steps and observe the rotation of the response potentiometer.
3. Record and tabulate Θ_R , V_R , Θ_0 and V_0 for a value of $K_A = 5$ as shown in Table 1.
4. Calculate the errors $(\Delta\Theta_R - \Delta\Theta_0)$, $(\Delta V_R - \Delta V_0)$ at each step.

Table 1 Manual Operation of the Position Control System

Set $K_D=0$, $K_A=5$

Sl.No.	θ_R deg	$\Delta\theta_R$ deg	θ_0 deg	$\Delta\theta_0$ deg	$\Delta\theta_R - \Delta\theta_0$ deg	V_R volts	V_0 volts	$\Delta V_R - \Delta V_0$ volts

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(b) Position Control through Step Command (without tachogenerator feedback)

1. Ensure that the motor unit is set to negative.
2. Set $K_A = 3$ and $K_D = 0$.
3. Connect the CRO and switch to CAPTURE mode.
4. Apply STEP input. Wait till the response is displayed. Trace the waveform displayed in CRO.
5. Compute M_p , t_s , t_p and the e_{ss} .
6. Repeat for $K_A = 4, 5, \dots$
7. Now $K_A = 5$ and choose various values of $K_D = 0.1, 0.2, \dots$ and repeat the above steps.
8. Tabulate the readings as shown in Table 2.

Table 2 Step Response of the Position Control (without Tachogenerator Feedback)

Set $K_D = 0$; $V_s = 2.5$ V (internally set)

Sl.No.	K_A	$M_p\%$	t_p (ms)	t_s (ms)	δ	e_{ss} (volts)	ω_n rad/sec

(c) Position Control through Step Command (with tachogenerator feedback)

1. Ensure that the tachogenerator feedback **switch is ON** and the motor unit is set to negative.
2. Now $K_A = 5$ and choose various values of $K_D = 0.1, 0.2, \dots$
3. Connect the CRO and switch to CAPTURE mode.
4. Apply STEP input. Wait till the response is displayed. Trace the waveform displayed in CRO.
5. Compute M_p , t_s , t_p and the e_{ss} .
6. Tabulate the readings as shown in Table 3.

Table 3 Step Response of the Position Control (with Tachogenerator Feedback)

Set $K_A = 7$; $V_s = 2.5$ volts (internally set)

Sl.No.	K_D	$M_p\%$	t_p (ms)	t_s (ms)	δ	e_{ss} (volts)	ω_n rad/sec

SPECIMEN CALCULATION

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RESULT

The steady state and transient response characteristics of a DC motor position control system were obtained by conducting the experiments as mentioned in the procedure.

From the calculated values of δ and ω_n obtained the closed loop transfer function for various values of K_A and K_D .

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Exp.No:

Date :

DESIGN AND IMPLEMENTATION OF A PHASE LEAD COMPENSATOR USING MATLAB SOFTWARE

AIM

1. To design a phase lead compensator for the given system and implement it using MATLAB software.
2. To compare the transient response of the uncompensated and compensated systems.

PROBLEM

Design a phase lead compensator for the system given by

$$G(s) = \frac{15}{s(s+1)} \text{ to satisfy the following specifications.}$$

SPECIFICATIONS

1. Phase margin ≥ 45 deg
2. e_{ss} for unit ramp input is $\leq 1/15$
3. ω_{gc} of the system must be less than 7.5 rad/sec

THEORY

In compensator design, gain adjustment is the first step to meet the required specification. The adjustment of gain alone will not be sufficient to meet the given specifications. In many cases, increasing the gain may result in poor stability or instability. In such cases, it is necessary to introduce additional devices or components in the system to alter the behaviour and to meet the desired specifications. Such a design or addition of a suitable device is called compensation. A device inserted into the system for the purpose of satisfying the specifications is called a compensator. The compensator is basically the introduction of a pole and/or zero in open loop transfer function to obtain the desired performance of the system. A compensator having the characteristics of a lead network is called a lead compensator. If a sinusoidal signal is applied to the lead network, then in steady

state the output will have a phase lead with respect to the input. Generally, lead compensation is provided to make an unstable system as a stable system.

DESIGN

The transfer function of the given uncompensated system is

$$G(s) = \frac{15}{s(s+1)}$$

To obtain the frequency response of the system, let $s = j\omega$,

$$G(j\omega) = \frac{15}{j\omega(j\omega+1)}$$

The magnitude and phase responses can be determined as described below.

MAGNITUDE PLOT

The corner frequency of the above system is $\omega_{c1} = 1$ rad/sec.

Choose a low frequency $\omega_l < \omega_{c1}$ and choose a high frequency $\omega_h > \omega_{c1}$.

Let $\omega_l = 0.1$ rad/sec

$\omega_h = 10$ rad/sec

Table 1 Magnitude plot for uncompensated System

Term	Slope (db/dec)	Change in slope (db/dec)
$\frac{15}{j\omega}$	-20 db/dec	
$\frac{1}{(j\omega+1)}$	-20db/dec	-40 db/dec

Let $M = |G(j\omega)|$ in db

1. At $\omega = \omega_l = 0.1$ rad/sec,

$$\begin{aligned} M &= 20 \log (15/j\omega) \\ &= 20 \log (15/0.1) \\ &= 43.52 \text{ db} \end{aligned}$$

2. At $\omega = \omega_{c1} = 1$ rad/sec,

$$\begin{aligned} M &= 20 \log (15/j\omega) \\ &= 20 \log (15/1) \\ &= 23.52 \text{ db} \end{aligned}$$

3. At $\omega = \omega_{c2} = 10$ rad/sec,

$$\begin{aligned} M &= \left[(\text{slope from } \omega_c \text{ to } \omega_h) \times \log \left(\frac{\omega_h}{\omega_c} \right) \right] + M|_{\omega=\omega_c} \\ &= -40 \log (10/1) + 23.52 \\ &= -16.48 \text{ db} \end{aligned}$$

PHASE PLOT

The phase angle of $G(j\omega)$ as a function of ω is given by

$$\Phi = -90^\circ - \tan^{-1} \omega$$

The phase angle of $G(j\omega)$ are calculated for various values of ω as shown in Table 2.

Table 2 Phase angle plot for uncompensated System

$\omega(\text{rad/sec})$	$\Phi(\text{deg})$
0.1	-95.71
0.5	-116.57
1.0	-135.00
2.0	-153.43
3.0	-161.57
4.0	-165.96
10	-174.29
15	-176.18
20	-177.13
50	-178.85
100	-179.42

The magnitude and phase angle plots are shown in Fig. 1. From the Bode plot of the uncompensated system, the phase margin is found to be 15° . But the system requires a

phase margin of 45° . So lead compensation should be employed to improve the phase margin of the system. The block diagram and transient response of the uncompensated system are shown in Fig. 2 and 3.

```

% uncompensated
num=[15];
den=[1 1 0];
w=logspace(-1,3);
[mag, phase, w]=bode(num, den, w);
[gm, pm, wpc, wgc]=margin(mag, phase, w);
figure(1);
bode(num, den, w);
margin(mag, phase, w);
    
```

AU

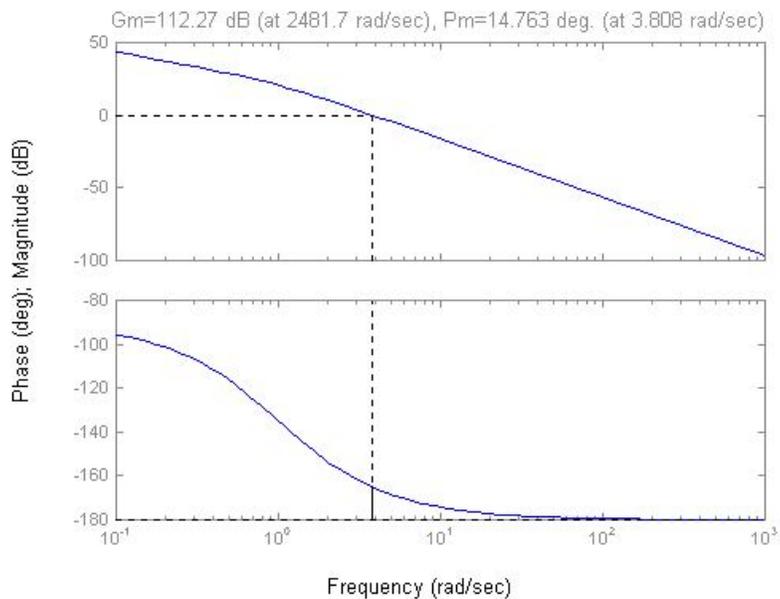


Fig. 1 Bode plot of uncompensated system

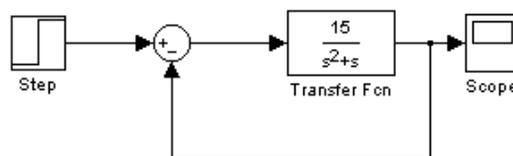


Fig. 2 Block diagram of uncompensated system

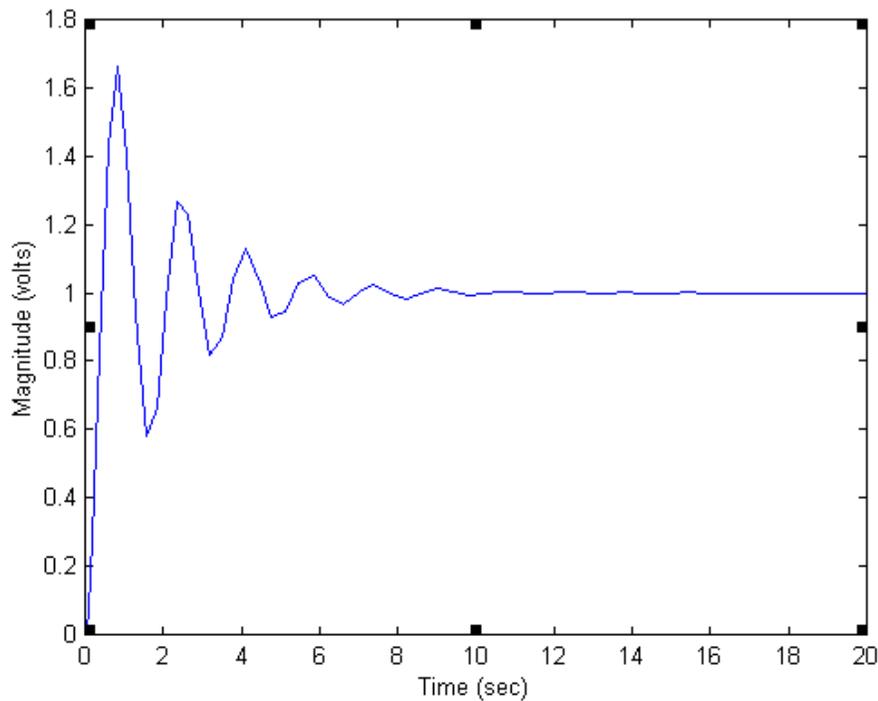


Fig. 3 Transient response of uncompensated system

Let Φ_d be the desired phase margin and it is given by

$$\Phi_m = \Phi_d - \Phi_u + \varepsilon$$

where

Φ_m - Maximum phase lead angle of the lead compensator.

Φ_d - Desired phase margin (45°)

Φ_u - Phase margin of the uncompensated system (15°)

ε - Additional phase lead to compensate for shift in gain crossover frequency.

(ε is assumed as 5°)

$$\begin{aligned} \Phi_m &= \Phi_d - \Phi_u + \varepsilon \\ &= 45^\circ - 15^\circ + 5^\circ = 35^\circ \end{aligned}$$

To determine the transfer function of lead compensator,

$$\alpha = \frac{1 - \sin \phi_m}{1 + \sin \phi_m}$$

$$\begin{aligned}\alpha &= \frac{1 - \sin 35^\circ}{1 + \sin 35^\circ} \\ &= 0.27\end{aligned}$$

From the bode plot, determine the frequency ω_m at which the magnitude of

$$G(j\omega) = -20 \log (1/\sqrt{\alpha})$$

For the given problem,

$$-20 \log (1/\sqrt{\alpha}) = -5.686 \text{ db}$$

Hence the frequency at this gain is found to be

$$\omega_m = 5.4 \text{ rad/sec}$$

Calculate τ from the relation between ω_m and α as given below.

$$\tau = \frac{1}{\omega_m \sqrt{\alpha}} = \frac{1}{5.4 \sqrt{0.27}} = 0.35 \text{ sec}$$

The transfer function of lead compensator is now written as

$$G(s) = \frac{\alpha(1 + \tau s)}{1 + \alpha \tau s} = \frac{0.27(1 + 0.35s)}{1 + 0.096s}$$

To implement this, an amplifier of gain $(1/\alpha)$ is to be considered. Therefore for this problem, the amplifier gain is chosen to be $\frac{1}{0.27} = 3.7$. Now the transfer function of the compensator alone is written as

$$G_c(s) = \frac{1 + 0.35s}{1 + 0.096s}$$

and the compensated system transfer function is finally given as

$$G(s) = \frac{15}{j\omega(j\omega + 1)} \frac{(1 + 0.35j\omega)}{(1 + 0.096j\omega)}$$

The magnitude and phase plot for the compensated system is now determined as follows.

MAGNITUDE PLOT

The corner frequencies are

$$\omega_1 = 0.1 \text{ rad/sec}$$

$$\omega_{c1} = 1/1 = 1 \text{ rad/sec}$$

$$\omega_{c2} = 1/0.096 = 10.416 \text{ rad /sec}$$

$$\omega_h = 50 \text{ rad / sec}$$

Table 3 Magnitude Plot for Compensated System

Term	Slope(db/dec)	Change in slope (db/dec)
$\frac{1}{j\omega}$	-20 db/dec	
$\frac{1}{j\omega + 1}$	-20db/dec	-40db/dec
$1+0.356 j\omega$	20 db/dec	-20 db/dec
$\frac{1}{1+0.096 j\omega}$	-20 db/dec	-40 db/dec

At $\omega = \omega_1 = 0.1 \text{ rad/sec}$, $M = 20 \log \left(\frac{15}{0.1} \right) = 43.52 \text{ db}$

At $\omega = \omega_{c1} = 1 \text{ rad/sec}$, $M = 20 \log \left(\frac{15}{1} \right) = 23.52 \text{ db}$

At $\omega = \omega_{c2} = 2.8 \text{ rad/sec}$, $M = -40 \log \left(\frac{2.8}{1} \right) + 23.52 = 5.26 \text{ db}$

At $\omega = \omega_{c3} = 10.416 \text{ rad/sec}$, $M = -20 \log \left(\frac{10.416}{2.8} \right) + 5.26 = -6.15 \text{ db}$

At $\omega = \omega_h = 50$ rad/sec, $M = -40 \log \left(\frac{50}{10.416} \right) - 6.15 = -33.4$ db

PHASE PLOT

$$\phi = -90^\circ - \tan^{-1}\omega - \tan^{-1} 0.096\omega + \tan^{-1} 0.356\omega$$

Table 4 Phase Plot for Compensated System

ω (rad/sec)	ϕ (deg)
0.1	-94.22
1	-120.88
2	-128.85
5	-133.65
10	-143.8
50	-170.3

From the Bode plot shown in Fig. 4, it is observed that the phase angle of the lead compensated system is improved to 45° as desired. Hence the designed compensator makes the overall systems to satisfy the given specifications. The block diagram and transient response of compensator system are shown in Fig. 5 and 6.

```

% compensated
num=[5.25 15];
den=[0.096 1.096 1 0];
w=logspace(-1,3);
[mag, phase, w]=bode(num, den, w);
[gm,pm,wpc,wgc]=margin(mag, phase, w);
figure(2);
bode(num, den, w);
margin(mag, phase, w);

```

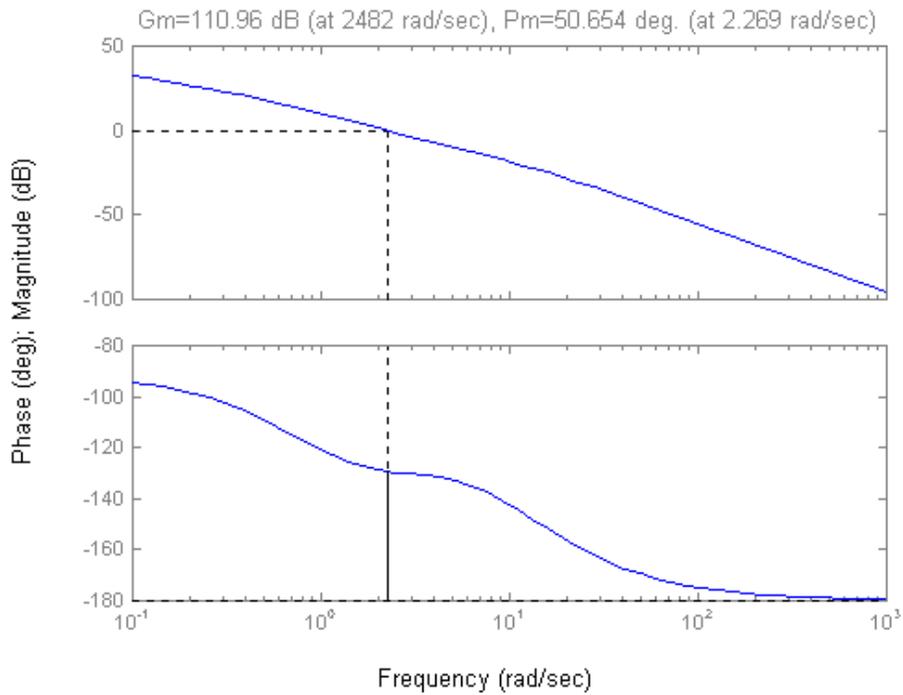


Fig 4 Bode plot of Compensated System

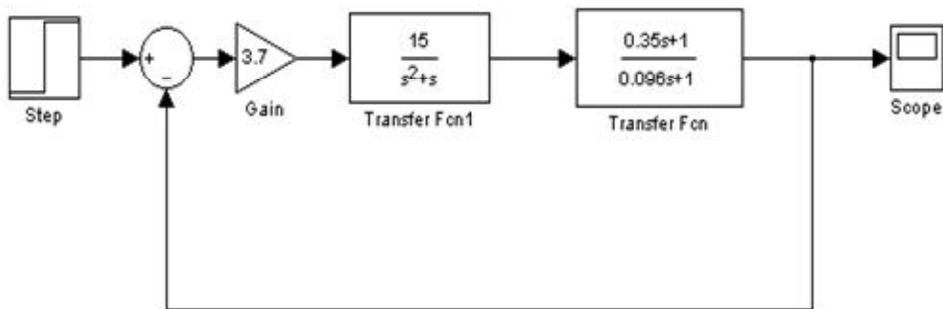


Fig. 5 Block diagram of compensated system

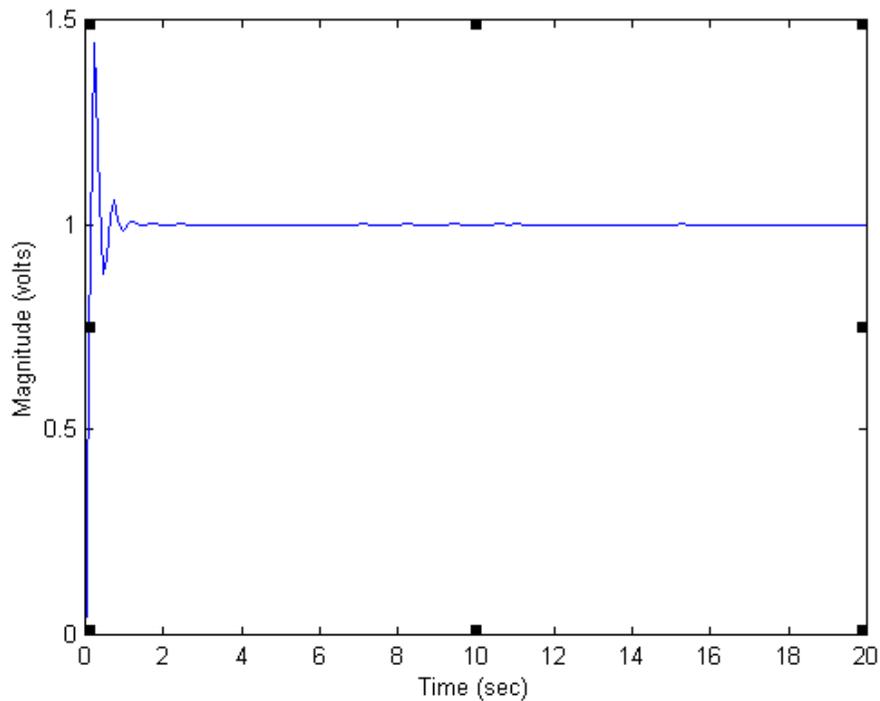


Fig. 6 Transient response of compensated system

RESULT

The phase lead compensator for the given system was designed and implemented using MATLAB Software.

Compare the obtained transient response parameters of uncompensated and compensated systems as shown in Fig.3 and Fig.6 and record your inference.

Exp.No:

Date :

IDENTIFICATION OF A GIVEN SYSTEM USING FREQUENCY RESPONSE CHARACTERISTICS

AIM

To identify the given system by obtaining the frequency response characteristics.

MAJOR APPARATUS REQUIRED

Hardware of given system

Signal generator

CRO

THEORY

Lead Network

When the sinusoidal input applied to a network produces a sinusoidal steady state output having a phase lead with respect to input then that network is called a lead network.

A lead compensating network is shown in Fig.1 (a).

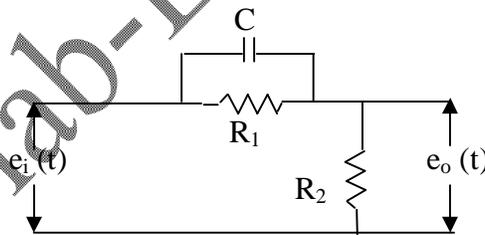


Fig. 1(a) Lead Network

The transfer function of the lead network is

$$G_{lead}(s) = \frac{\alpha(1 + \tau s)}{(1 + \alpha \tau s)} \quad \text{Where } \tau = R_1 C \quad \alpha = \frac{R_2}{R_1 + R_2} < 1$$

The parameters α and τ can be determined from frequency response characteristics

as $\tau = \frac{1}{\omega_{c1}}$, $\alpha\tau = \frac{1}{\omega_{c2}}$ and $20\log\frac{1}{\alpha} = \omega_m$.

The frequency response of the lead network is shown in Fig. 1(b)

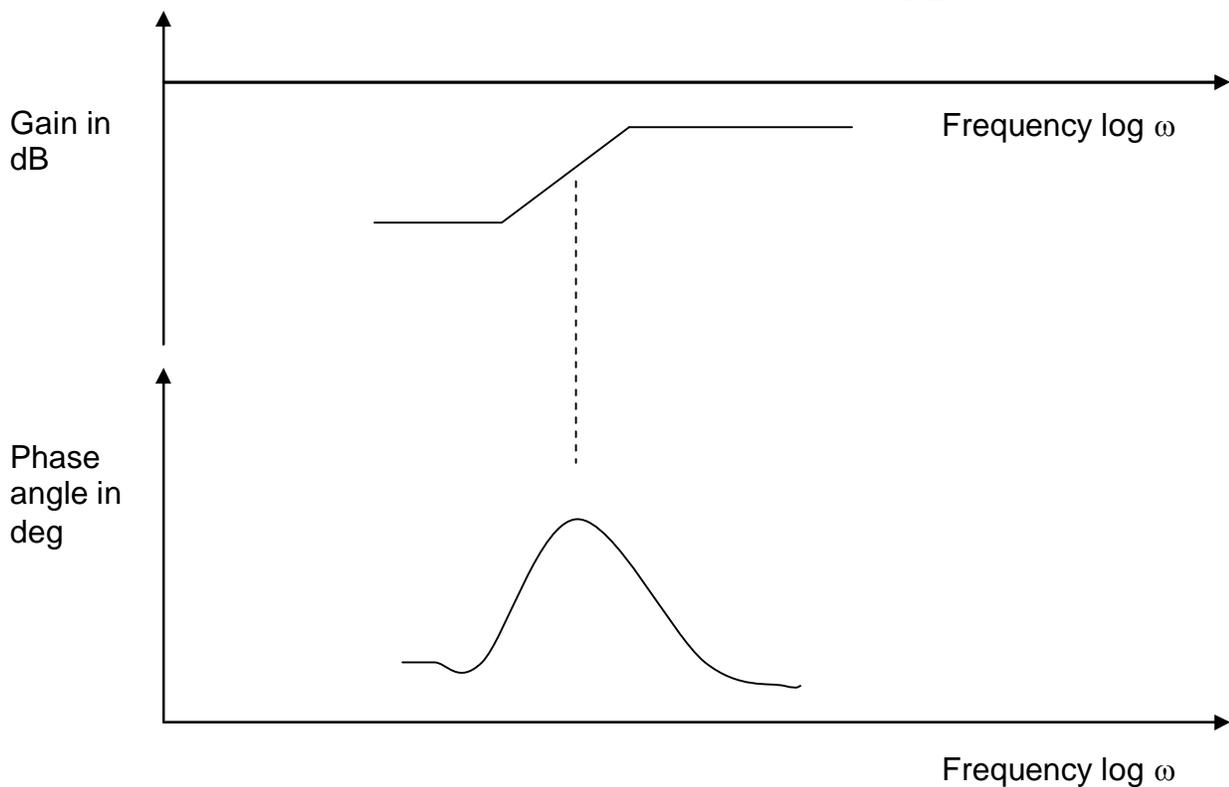


Fig. 1(b) Frequency response of a Lead Network

Lag Network

When the sinusoidal input applied to a network produces a sinusoidal steady state output having a phase lag with respect to input then that network is called a lag network.

A lag compensating network is shown in Fig. 2(a).

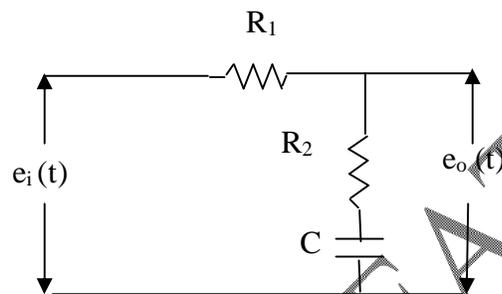


Fig. 2 (a) Lag Network

The transfer function of the lag network is

$$G_{lag}(s) = \frac{\beta(1 + \tau s)}{(1 + \beta\tau s)}$$

Where $\tau = R_2 C$

$$\beta = \frac{R_1 + R_2}{R_2} > 1$$

The parameters α and β can be determined from frequency response characteristics

$$\text{as } \tau = \frac{1}{\omega_{c1}}, \beta\tau = \frac{1}{\omega_{c2}} \text{ and } 20 \log \frac{1}{\beta} = \omega_m.$$

The frequency response of the lag network is shown in Fig. 2(b)

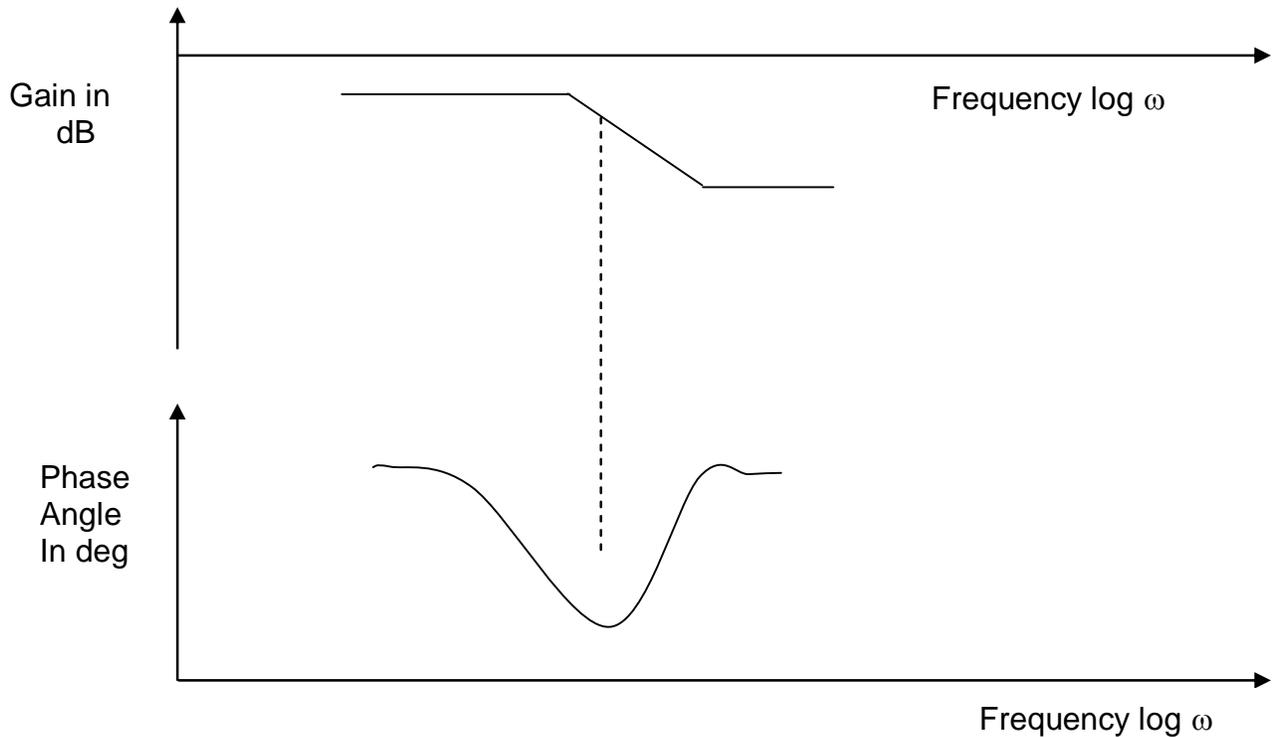


Fig. 2(b) Frequency response of a Lag network

PROCEDURE

To identify the given system, its frequency response a characteristic is determined as mentioned below.

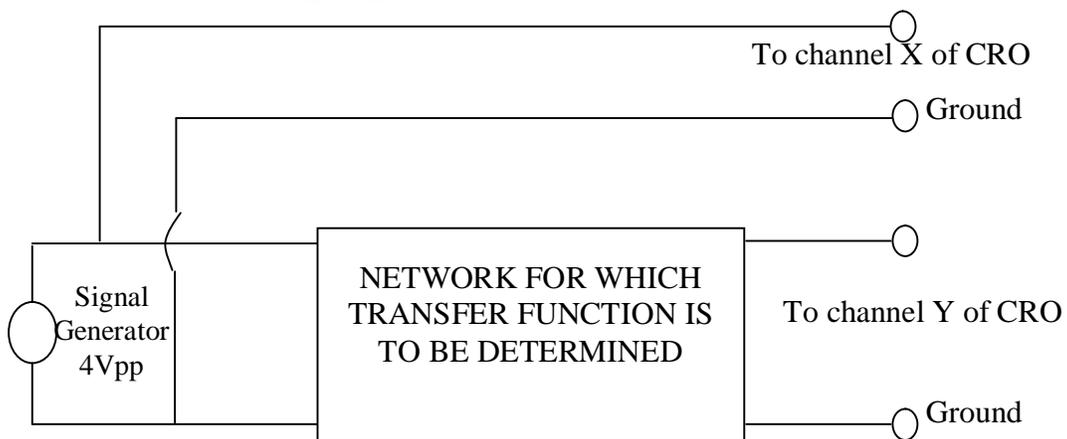
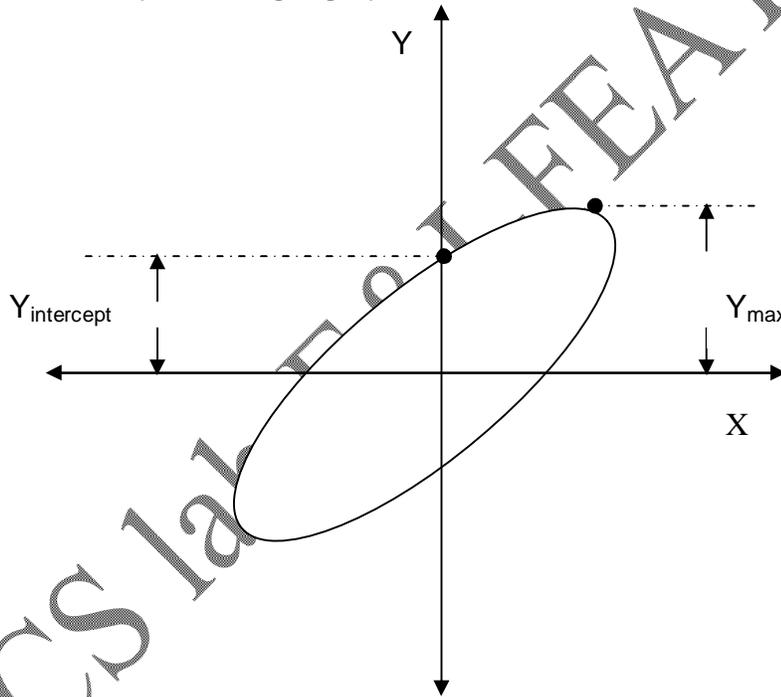


Fig.3 Circuit Diagram for Frequency Response Characteristics

1. Make connections as shown in Fig. 3.
2. Set the magnitude of sinusoidal input signal from the signal generator at 4Vpp.
3. Set the CRO time scale at 1/10th of the frequency of next higher scale.
4. Set the magnitude scale at appropriate position.
5. Vary the frequency of the input signal from 50Hz to 100 KHz and note down voltage at the output.
6. To measure the phase difference between input and output, put the CRO in XY mode and determine the phase angle. The phase angle can be determined as shown in Fig.4.
7. Tabulate the readings and compute the gain in dB. Using a semilog sheet draw the magnitude and phase angle graph.



$$\text{Phase angle } (\phi) = \text{Sin}^{-1} \left(\frac{Y_{\text{intercept}}}{Y_{\text{maximum}}} \right)$$

Table 1 Frequency Response

SI. NO	FREQUENCY IN Hz	V _o IN VOLTS	GAIN IN dB	Y INTERCEPT	Y MAX	PHASE ANGLE (ϕ)

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DETERMINATION OF TRANSFER FUNCTION

1. Compare the obtained frequency response with the frequency response of a lead network and that of a lag network to identify whether the given system is a lead or a lag network.

$$G_{\text{lead}}(s) = \frac{\alpha(1 + \tau s)}{(1 + \alpha\tau s)} \quad \text{and} \quad G_{\text{lag}}(s) = \frac{\beta(1 + \tau s)}{(1 + \beta\tau s)}$$

2. Determine ω_{c1} which is the frequency at which the gain tends to increase sharply. $\frac{1}{\omega_{c1}}$ gives the value of τ (time constant).
3. Calculate the value of α from low frequency gain (constant gain region). $20 \log \alpha = GL$ (Gain at Low frequency) or Calculate the value of β from low frequency gain (constant gain region). $20 \log \beta = GL$ (Gain at Low frequency)
4. Determine ω_{c2} , the frequency at which the Bode plot deviates from mid frequency linear region.

RESULT

1. Thus the given network is identified as ----- network. The transfer function of the network was determined from the obtained frequency response characteristics. The parameters of the network were identified as

$$R_1 =$$

$$R_2 =$$

$$C = \quad \quad \quad (\text{Assumed}).$$

2. Thus the transfer function of the given system can be written as

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Exp.No:
Date :

(A) CHARACTERISTICS OF SAMPLE AND HOLD CIRCUIT

AIM

To determine the characteristics of sample and hold IC LF398.

MAJOR APPARATUS REQUIRED

Signal generator
Regulated Power Supply
Sample and Hold circuit

NEED FOR SAMPLE AND HOLD CIRCUIT

As the complexity of a control system increases, the cost of an analog controller rises steeply. In fact, constructing a complex control function may even become technically infeasible, if one is restricted to use only analog elements. The use of digital controller has grown tremendously with less cost and with improved reliability. Digital controller used in digital control system have the inherent characteristics that they accept the data as short duration pulses and produce a similar kind of output as control signals.

A sampler and analog-to-digital converter (ADC) are needed at the computer input. The sampler converts the continuous time signal into a sequence of pulses. The output data of digital computer (controller) are converted into continuous time signal by digital-to-analog converter (DAC) and a hold circuit. The overall system as shown in Fig.1 is hybrid in which, the signal is in sampled form in the digital controller and in continuous form in the rest of the system. A system of this kind is referred to as a sampled-data system and the knowledge about sample and hold circuit becomes essential.

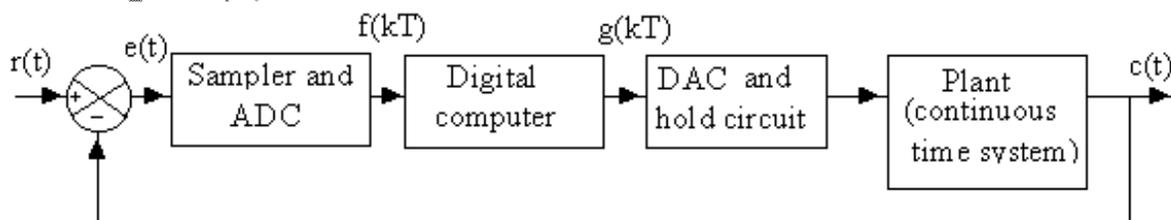


Fig. 1 Block diagram of sampled data system

Sampled data technique is more appropriate for control systems requiring long distance data transmission. Signal sampling reduces the power demand made on the signal and is therefore helpful for signals of weak power origin. The circumstances that lead to the use of sampled-data control system are summarized as follows:

1. For using digital computer (or microprocessor) as part of the control loop.
2. For time-sharing of control components.
3. Whenever a transmission channel forms part of the control loop.
4. Whenever the output of a control component is essentially in discrete form

Sampling implies that the signal at the output of the sampler is available in the form of short duration pulses each followed by a skip period when no signal is available so that the control system essentially operates open loop during the skip period. It is intuitively obvious that if the sampling rate (or frequency) is too low, significant information contained in the input signal may be missed in the output. This may be overcome by using the hold circuit.

Hence, the sampling rate is chosen such that it should satisfy the Shannon's sampling theorem. According to this theorem, the information contained in a signal is fully preserved in the sampled version so long as the sampling frequency is at least twice the maximum frequency contained in the signal.

Both the sample and hold operations can effectively be achieved by implementing IC LF398. The LF398 is a monolithic sample and hold circuit which utilizes JFET technology to obtain ultra high DC accuracy of 0.002% with fast acquisition of signal at the rate of 6 microseconds and with low droop rate.

PRECAUTIONS

The sample and hold circuit IC is very sensitive to voltage fluctuations. Observe the following precautions before switching on the power supply.

1. Switch ON the power supply, by keeping the voltage initially at zero.
2. Now increase the voltage gradually from 0 to +12 V and – 12 V in the dual supply.

EXPERIMENTAL PROCEDURE

To verify the performance of sample and hold circuit

1. Make the connections as per the circuit diagram shown in Fig.2.
2. Keep the signal frequency as 50 Hz and sampling signal frequency (logic input) at multiples of 50 Hz like 100 Hz, 150 Hz, 200 Hz.
3. Switch ON the power supplies and signal generator, Trace the sample and hold circuit output (model waveforms as shown in Fig.3).
4. Document the inferences observed on the output waveforms.

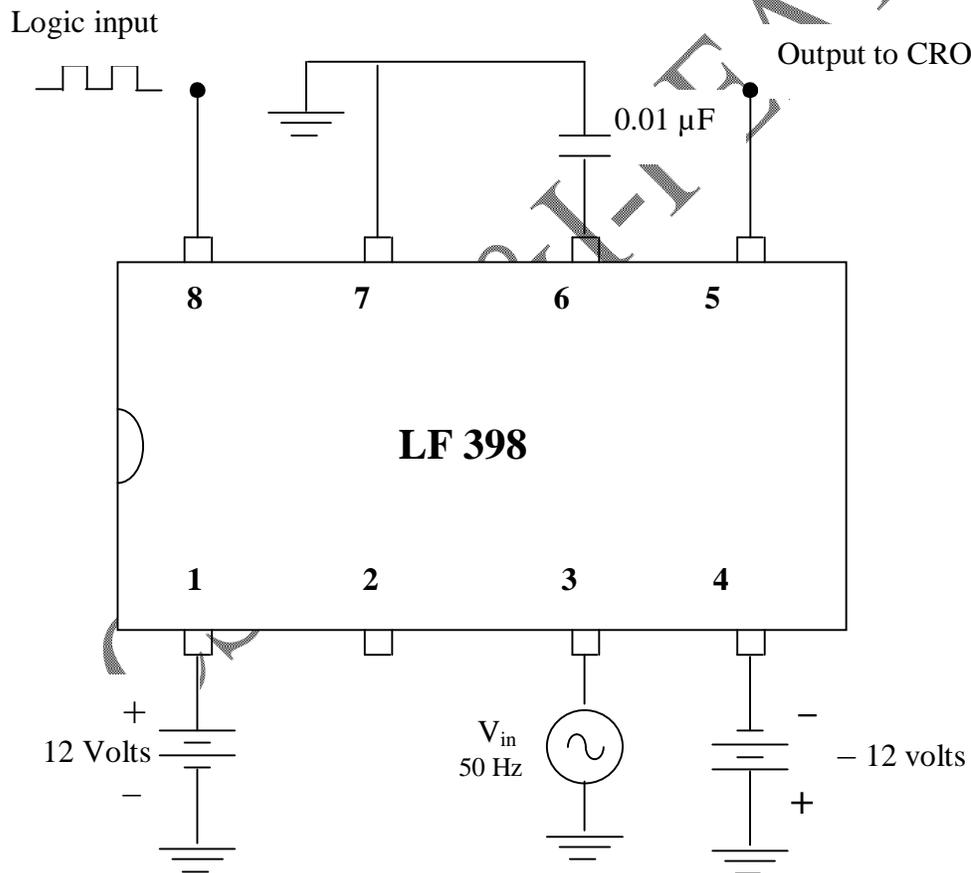


Fig. 2 Circuit diagram of LF398

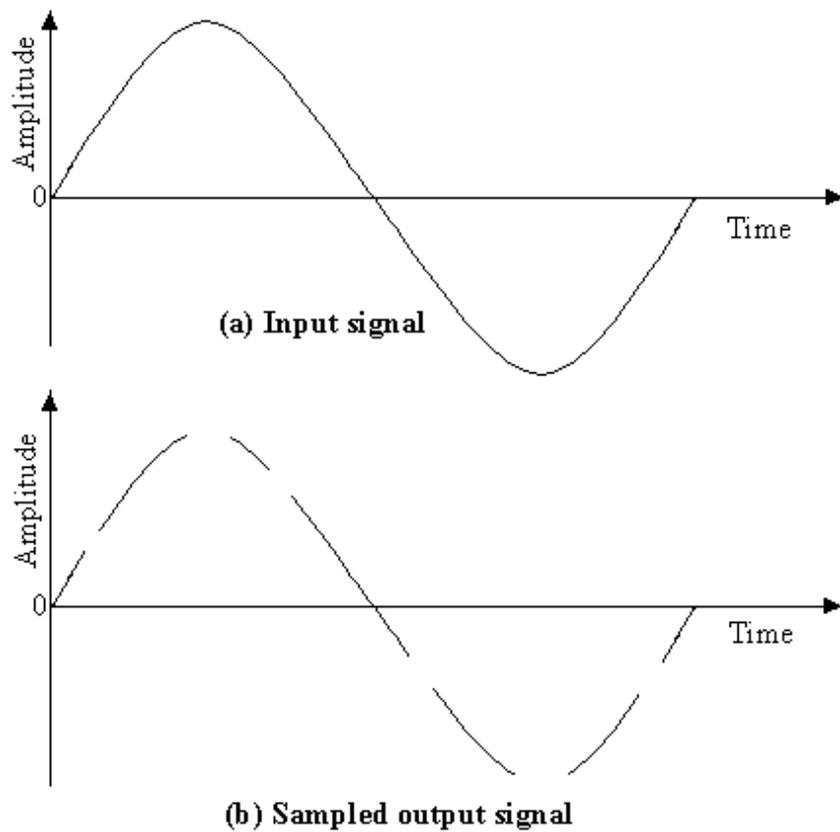


Fig. 3 Model Waveforms of Sample and Hold IC

RESULT

The characteristics of the sample and hold IC were studied.

(B) SIMULATION OF A SAMPLED DATA CONTROL SYSTEM

AIM

To develop a difference equation representation of a sampled data system from the transfer function model and to obtain a unit step response of the model using digital computer.

DIFFERENCE EQUATION MODEL

The difference equation representation enables us to analyze sampled-data system shown in Fig.1. The difference equation is an approximation of the differential equation that represents a continuous-time system. However it would yield a highly erroneous solution if the sampling rate (T) is not properly chosen.

The general form of n^{th} -order linear, constant coefficient difference equation (LCCDE) is given by

$$y(k+n) + a_1 y(k+n-1) + \dots + a_n y(k) = b_0 r(k+m) + b_1 r(k+m-1) + \dots + b_m r(k) \quad (1)$$

Often this equation is written in the equivalent form as:

$$y(k) + a_1 y(k-1) + \dots + a_n y(k-n) = b_0 r(k) + b_1 r(k-1) + \dots + b_m r(k-m) \quad (2)$$

The coefficients a_i and b_j are real constants, k , m and n are integers with $m \leq n$, $i = 1, \dots, n$ and $j = 1, \dots, m$. It may be noted that the difference equation solution yields $y(k)$, the values of output $y(t)$ at sampling instants only. No information is available about $y(t)$ during sampling instants. This is satisfactory so long as the sampling rate is sufficiently fast in order that any important output information is not missed. Therefore it can be concluded that the sampled-data system is conditionally stable but continuous time system is absolutely stable. The instability of the sampled-data system results from the fact that it operates open-loop between sampling instants.

The equation (2) is iteratively solved with past values of outputs: $y(-1), y(-2), y(-3) \dots y(-n)$ and present input $r(0)$ to get the output at instant '0'. If 'k' is incremented to take on values $k = 0, 1, 2 \dots \text{etc.}$, then corresponding $y(k)$ is generated.

Let the given sampled data system be represented as

$$\frac{y(s)}{e(s)} = \frac{k_p}{s(s+a)}; \text{ where } k_p \text{ and 'a' are constants.} \quad (3)$$

$$(s^2 + as)y(s) = k_p e(s)$$

$$s^2 y(s) + a s y(s) = k_p (r(s) - y(s))$$

This can be written as

$$\ddot{y}(t) + a\dot{y}(t) + k_p y(t) = k_p r(t) \quad (4)$$

Replace t by kT in the above equation in order to get difference equation model as follows:

$$\frac{[\dot{y}(k+1) - \dot{y}(k)]}{T} + a \frac{[y(k+1) - y(k)]}{T} + k_p y(k) = k_p r(k) \quad (5)$$

$$[\dot{y}(k+1) - \dot{y}(k)] + a[y(k+1) - y(k)] + T k_p y(k) = T k_p r(k) \quad (6)$$

$$\frac{[y(k+2) - y(k+1)]}{T} - \frac{[y(k+1) - y(k)]}{T} + a[y(k+1) - y(k)] + T k_p y(k) = T k_p r(k)$$

$$[y(k+2) - y(k+1)] - [y(k+1) - y(k)] + aT[y(k+1) - y(k)] + T^2 k_p y(k) = T^2 k_p r(k)$$

After simplification the above equation becomes

$$y(k+2) = T^2 k_p r(k) + (2 - aT)y(k+1) - (1 - aT + T^2 k_p)y(k) \quad (7)$$

The above difference equation model is solved for a unit step input ($r(k)=1$) using Program in C-language and the corresponding output $y(k)$ is plotted as shown in model waveforms in Fig. 4.

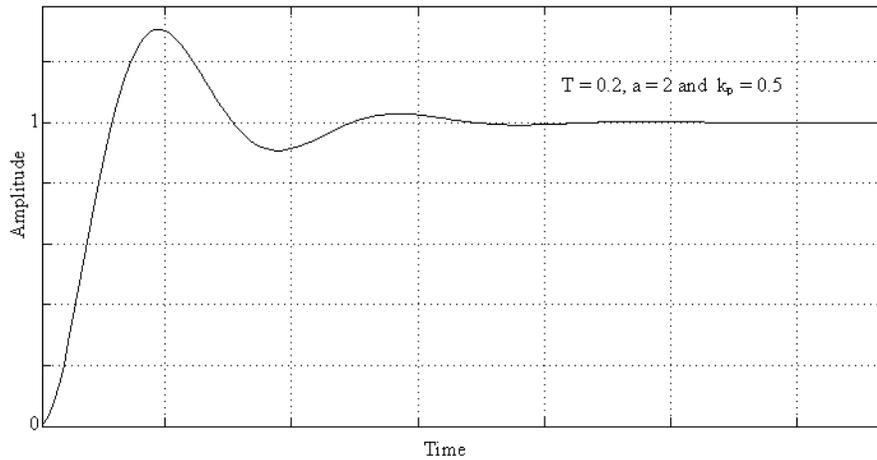


Fig. 4 Unit Step response of Sampled data System

C Program

```
# include <stdio.h>
# include<conio.h>
# include<graphics.h>
main ( )
{
  int k;
  float kp , T,a, y[300];
  int m=0, n=0;
  clrscr( );
  initgraph (&m,&n, "");
  printf ("enter the values of T,a, kp");
  scanf(" %f%f%f",&T,&a,&kp);
  y[0] = 0;
  y[1] = 0;
  for (k=0;k<=300;k++)
  {
    y[k+2] = (kp * T*T* r[k]) + (y[k+1] *(2- a*T))-( y[k]* (1- a*T+kp*T*T));
    putpixel (k, 300-y[k]*100,15);
  }
  getch ( );
}
```

RESULT

The difference equation model was developed for the given sampled-data system and output response for unit step input was obtained.

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Exp.No:

Date :

(A) SENSITIVITY ANALYSIS OF OPEN LOOP AND CLOSED LOOP SYSTEMS USING PROCESS CONTROL SIMULATOR

(B) STABILITY CHARACTERISTICS OF FEEDBACK SYSTEMS USING PROCESS CONTROL SIMULATOR

AIM

1. To determine the effect of feedback on
 - (a) Sensitivity of the system to change in gain
 - (b) Speed of the response of the system
2. To determine the stability of the given Type 0 closed loop system.

(1) CHARACTERISTICS OF OPEN LOOP SYSTEMS

The open loop transfer function is given by

$$G(s) = \frac{K}{s+1}$$

CASE: I

$$G(s) = \frac{K}{s+1}$$

$$\text{Gain } K = \frac{100}{\%PB}$$

$$C(s) = G(s)R(s)$$

For $k=2$, Proportional Band (PB)=50%

$$G(s) = \frac{C(s)}{R(s)} = \frac{2}{s+1}$$

Since, $r(t)=2$, $R(s)=2/s$

$$C(s) = \frac{2}{s+1} * \frac{2}{s} = \frac{4}{s(s+1)}$$

Using partial fraction method,

$$C(s) = 4 \left[\frac{1}{s} - \frac{1}{s+1} \right]$$

Taking inverse laplace transform,

$$C(t) = 4(1 - e^{-t})$$

At steady state,

$$C_{ss}(t) = 4$$

CASE: II

$$G(s) = \frac{K}{s+1}$$

$$\text{Gain } K = \frac{100}{\%PB}$$

$$C(s) = G(s)R(s)$$

For $k=1$, Proportional Band (PB)=100%

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1}$$

Since, $r(t)=2$, $R(s)=2/s$

$$C(s) = \frac{1}{s+1} * \frac{2}{s} = \frac{2}{s(s+1)}$$

Using partial fraction method,

$$C(s) = 2 \left[\frac{1}{s} - \frac{1}{s+1} \right]$$

Taking inverse laplace transform,

$$C(t) = 2(1 - e^{-t})$$

At steady state,

$$C_{ss}(t) = 2$$

For open loop system when K=1, C=2V

when K=2, C=4V

$$\text{Sensitivity } y(S) = \frac{\text{Change in output / output}}{\text{Change in gain / gain}} = \frac{\Delta C / C}{\Delta K / K}$$

$$S = \frac{\frac{(4-2)}{2}}{\frac{(2-1)}{1}} = 1$$

CHARACTERISTICS OF CLOSED LOOP SYSTEMS

The closed loop transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) = \frac{K}{s+1}$$

Assuming unity feedback, H(s)=1

CASE: I

For k=2, Proportional Band (PB)=50%

The closed loop transfer function is given by,.

$$\frac{C(s)}{R(s)} = \frac{\frac{2}{s+1}}{1 + \frac{2}{s+1}}$$

$$G(s) = \frac{2}{s+3}$$

Since, $r(t)=2$, $R(s)=2/s$

$$C(s) = \frac{2}{s+3} * \frac{2}{s} = \frac{4}{s(s+3)}$$

Using partial fraction method,

$$C(s) = 4 \left[\frac{1}{3s} - \frac{1}{3(s+3)} \right]$$

$$C(s) = \frac{4}{3} \left[\frac{1}{s} - \frac{1}{(s+3)} \right]$$

Taking inverse laplace transform,

$$C(t) = \frac{4}{3} (1 - e^{-3t})$$

At steady state,

$$C_{ss}(t) = \frac{4}{3} = 1.33$$

CASE: II

For $k=1$, Proportional Band (PB)=100%

The closed loop transfer function is given by,.

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s+1}}{1 + \frac{1}{s+1}}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s+2}$$

Since, $r(t)=2$, $R(s)=2/s$

$$C(s) = \frac{1}{s+2} * \frac{2}{s} = \frac{2}{s(s+2)}$$

Using partial fraction method,

$$C(s) = \left[\frac{1}{s} - \frac{1}{(s+2)} \right]$$

Taking inverse laplace transform,

$$C(t) = (1 - e^{-2t})$$

At steady state,

$$C_{ss}(t) = 1$$

For close loop system when $K=1$, $C=1V$

when $K=2$, $C=1.33V$

$$\text{Sensitivity } (S) = \frac{\text{Change in output / output}}{\text{Change in gain / gain}} = \frac{\Delta C / C}{\Delta K / K}$$

$$S = \frac{\frac{(1.33 - 1)}{1}}{\frac{1}{(2 - 1)}} = 0.33$$

PROCEDURE

1.(a) Effect of feedback on Sensitivity

1. Give the connections as per the circuit diagram shown in Fig.1.
2. Set the process for 1 lag term, since the transfer function is $1/(s+1)$.
3. Set all the time switches at 1 sec, set $K=1$ i.e.PB=100%.
4. Adjust the set value to 2V.
5. Measure the output through a DMM.
6. Repeat the above procedure for $K=2$ and determine the sensitivity.
7. Repeat the above procedure for the closed loop condition by giving connections as shown in Fig.2
8. Determine the sensitivity using the formula.
9. Tabulate the observations as shown in Table 1.

1.(b) Effect of feedback on Speed of Response

1. Make connections as shown in Fig.1.
2. Set the gain as one and include one lag term to simulate the process given by

$$G(s) = \frac{1}{s+1}$$

3. Apply 2V step input and trace the response at output using CRO.
4. From the step response, determine the time constant (τ) by finding the time taken to reach 63.2% of final steady state value.
5. Repeat the above procedure for closed loop condition by making the connections as shown in Fig.2.
6. Compare the time constants under open loop and closed loop condition and write the observations as shown in Table2.

Observation

Open loop	Closed loop
1. More Sensitive	Less Sensitive
2. Slower Response	Faster Response

Table1 Sensitivity Measurement

PROCESS	GAIN(K)	OPEN LOOP		CLOSED LOOP	
		Theoretical	Practical	Theoretical	Practical
$\frac{K}{s+1}$ (First Order, Type 0)	1	C=2V		C=1V	
	2	C=4V		C=1.33V	
		S=1		S=0.345	

Table2 Observations of Speed of Response

PROCESS	GAIN(K)	Time Constant (τ) secs			
		OPEN LOOP		CLOSED LOOP	
		Theoretical	Practical	Theoretical	Practical
$\frac{K}{s+1}$ (First Order, Type 0)	1	0.99		0.499	

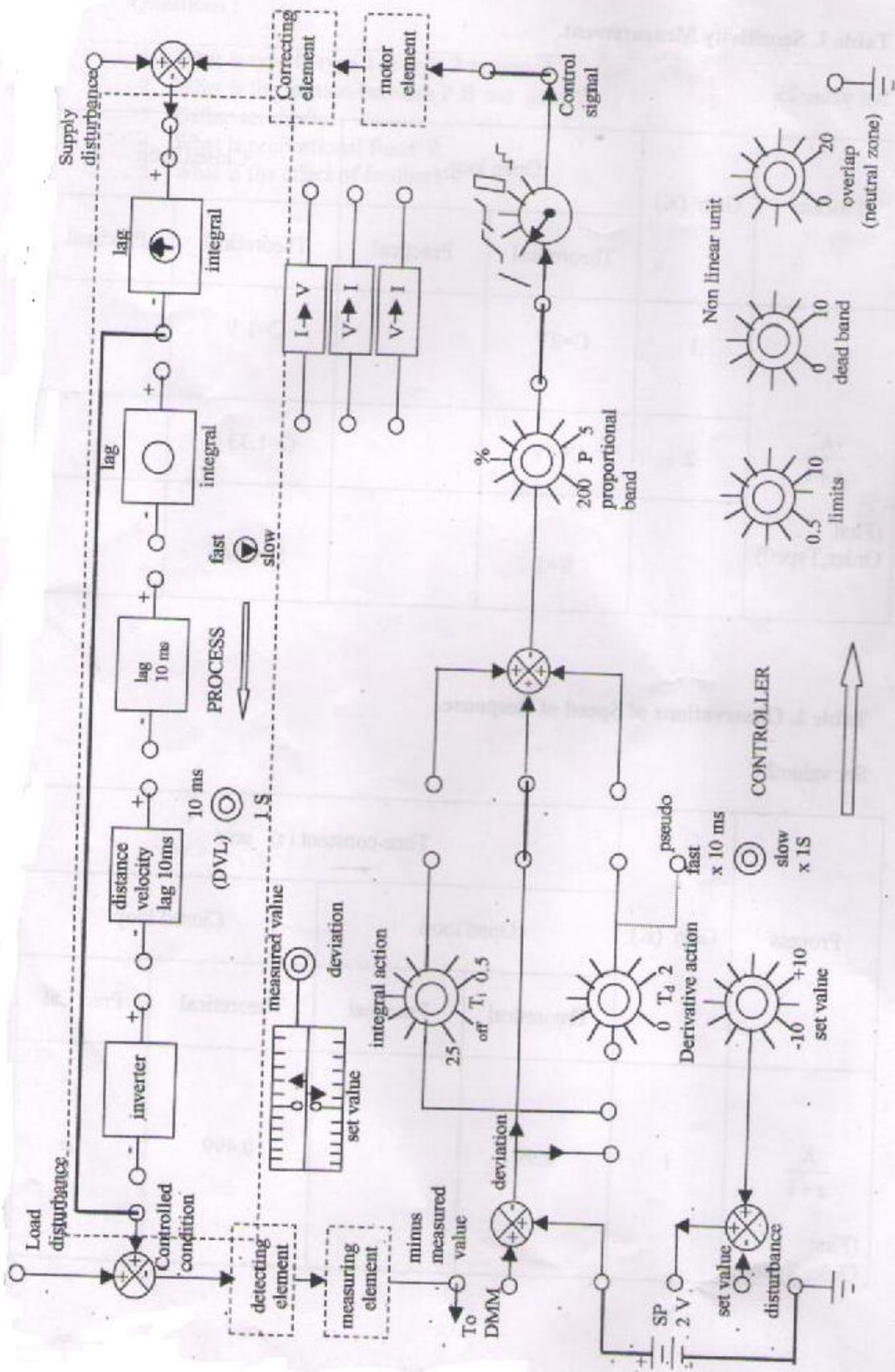


Fig 1. OPEN LOOP CONNECTION DIAGRAM

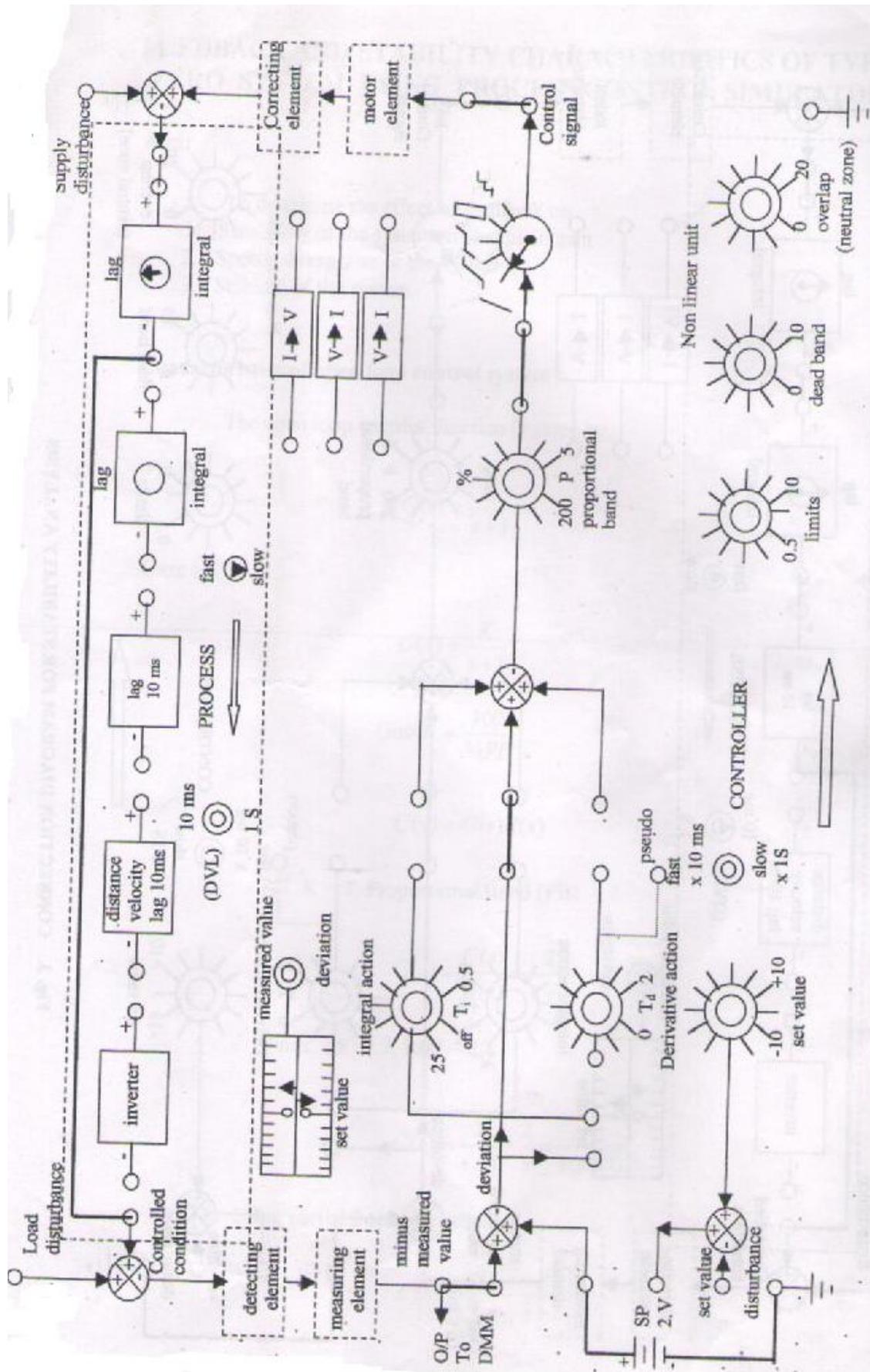


Fig.2.- CLOSED LOOP CONNECTION DIAGRAM

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2. Stability Characteristics

Consider Type 0 system as given below

$$G(s) = \frac{K}{(1 + 0.01s)^3}$$

$$G(s) = \frac{K}{(0.01)^3 (s + 100)^3}$$

$$G(s) = \frac{K_1}{(s + 100)^3}$$

Where

$$K_1 = 10^6 * K$$

The Characteristic equation is given by

$$1 + G(s)H(s) = 0; \text{ where } H(s) = 1$$

$$1 + \frac{K_1}{(s + 100)^3} = 0$$

$$s^3 + 300s^2 + 3 * 10^4 s + 10^6 + K_1 = 0$$

Apply Routh –Hurwitz criteria as given below

s^3	1	$3 * 10^4$
s^2	300	$10^6 + K_1$
s^1	$\frac{(9 * 10^6) - (10^6 + K_1)}{300}$	
s^0	$10^6 + K_1$	

$$(9 * 10^6) - (10^6 + K_1) = 0$$

$$10^6(9 - 1) - K_1 = 0$$

$$8 * 10^6 - K_1 = 0$$

$$\text{But } K_1 = K * 10^6$$

$$10^6(8 - K) = 0$$

$$K = 8$$

The system tends to be unstable when $K \leq 8$.

Frequency of oscillation

$$300s^2 + 10^6 + K_1 = 0$$

$$300s^2 + 10^6 + 8 * 10^6 = 0$$

Put $s = j\omega$

solve

$$\omega = 173.2 \text{ rad/sec.}$$

$$2\pi f = 173.2$$

$$f = 27.56 \text{ Hz.}$$

$$T = \frac{1}{f} = \frac{1}{27.56} = 0.036 \text{ sec}$$

$$T = 36 \text{ msec.}$$

PROCEDURE

1. Set the process for 3 lag terms since $G(s) = \frac{K}{(1+0.01s)^3}$ and make the connections as shown in Fig.3.
2. Set the time switches to 10 msec, since the coefficient of 's' term in the transfer function is 0.01.
3. Ground the setpoint terminals.
4. Set the CRO voltage scale at 2V/cm.
5. Slowly adjust the gain till a sustained oscillation is obtained.
6. Note the critical point for the system to be unstable.

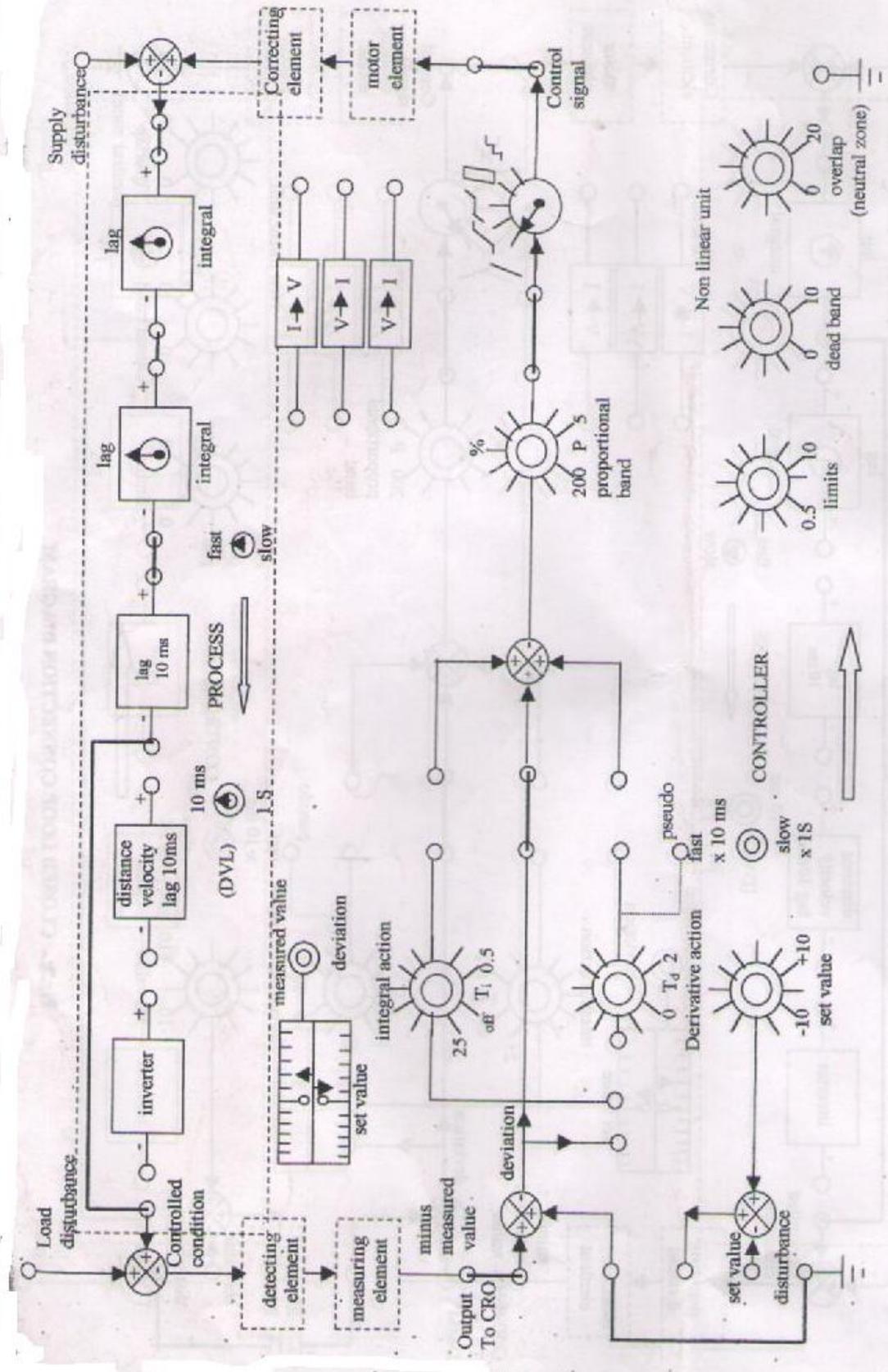


Fig. 3. CONNECTION DIAGRAM FOR STABILITY ANALYSIS

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RESULT

The effect of feedback on sensitivity, speed of response and stability of the system were studied.

Exp.No:

Date :

TIME RESPONSE ANALYSIS OF A SECOND ORDER TYPE-0 AND TYPE-1 SYSTEM USING PROCESS CONTROL SIMULATOR

AIM

To obtain the transient response of second order type-0 and type-1 systems using Process Control Simulator (PCS) and to determine the transient parameters namely peak overshoot, settling time, peak time and steady state parameters (steady state error).

MAJOR APPARATUS REQUIRED

Process Control Simulator

Digital Multimeter

Regulated Power Supply

CRO

THEORY

The dynamic behaviour of a system under the application of a standard test signal is termed as transient response of a system. The transient nature of the system is defined by the parameters such as peak time, settling time and peak overshoot. Some of the standard test signals are impulse, step, constant ramp (velocity) and constant parabolic (acceleration). The steady state response is characterized by the parameters such as steady state value and steady state error.

For a unity feedback system ($H(s) = 1$), the closed loop transfer function is given by,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{p(s)}{q(s)}$$

The closed loop transfer function of a standard second order system is

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

where

δ - Damping factor (or) Damping ratio

ω_n - Undamped natural frequency

For a unit step input, the output response of a second order system is given as

$$c(t) = 1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin \left[\omega_n \sqrt{1-\delta^2} t + \tan^{-1} \left(\frac{\sqrt{1-\delta^2}}{\delta} \right) \right]$$

The steady state value of the output $c(t)$ is given by

$$c_{ss} = \lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sC(s)$$

(i) Consider an open loop transfer function of the form

$$G(s) = \frac{K}{(s+1)(s+1)}$$

This open loop transfer function represents a type-0 system as there are no poles at the origin.

(ii) Consider an open loop transfer function of the form

$$G(s) = \frac{K}{s(s+1)}$$

This open loop transfer function represents a type-1 system as there is a pole at the origin.

TIME RESPONSE SPECIFICATIONS

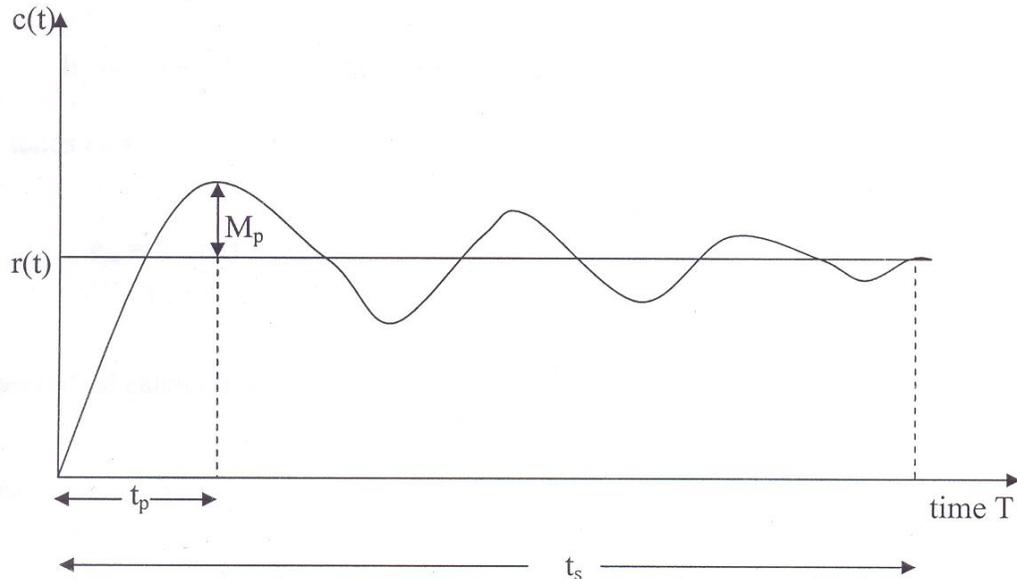


Fig.1. Time response of a Second order System

(a) Peak time

It is the time required for the response to reach the peak of time response or the peak overshoot. Peak time is given as

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}}$$

(b) Peak overshoot

It indicates the normalized difference between the peak of the time response and the steady state output. It is defined as

$$\%M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

(c) Settling time

It is the time required for the response to reach and stay within a specified tolerance band (usually 2% or 5%) of its final value and is given as

$$t_s = \frac{4}{\delta\omega_n} \quad (\text{tolerance band 2\%})$$

$$t_s = \frac{3}{\delta\omega_n} \quad (\text{tolerance band 5\%})$$

(d) Steady state error

It indicates the error between the actual output and the desired output as time, t tends to ∞ .

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)H(s)}$$

THEORETICAL CALCULATIONS

Case (i) Type-0 system with gain $K = 2$

$$G(s) = \frac{2}{(\tau s + 1)(\tau s + 1)} \quad \text{where } \tau = 1 \text{ sec}$$

The closed loop transfer function for this system is,

$$\frac{C(s)}{R(s)} = \frac{\frac{2}{(s+1)(s+1)}}{1 + \frac{2}{(s+1)(s+1)}} = \frac{2}{s^2 + 2s + 3}$$

Rearranging the above equation to a general second order form, then

$$\frac{C(s)}{R(s)} = \frac{2}{3} \left(\frac{3}{s^2 + 2s + 3} \right)$$

The general second order closed loop transfer function for type-0 system,

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Comparing the above two equations, the following parameters are obtained.

Natural frequency $\omega_n^2 = 3$;

$$\begin{aligned}\omega_n &= \sqrt{3} \\ &= 1.732 \text{ rad/sec}\end{aligned}$$

Damping factor $2\delta\omega_n = 2$;

$$\begin{aligned}\delta &= \frac{1}{\omega_n} \\ &= \frac{1}{\sqrt{3}} \\ &= 0.577\end{aligned}$$

Steady state value of the given system for step input is given by

$$c_{ss} = \lim_{s \rightarrow 0} sG(s)$$

We know that $\frac{C(s)}{R(s)} = \frac{2}{3} \left(\frac{3}{s^2 + 2s + 3} \right)$

For a step input of 2volts, $R(s) = \frac{2}{s}$.

$$C(s) = \frac{2}{s} \left(\frac{2}{3} \right) \left(\frac{3}{s^2 + 2s + 3} \right)$$

Applying Final Value Theorem,

$$c_{ss} = \lim_{s \rightarrow 0} \left(s \frac{2}{s} \left(\frac{2}{3} \right) \left(\frac{3}{s^2 + 2s + 3} \right) \right)$$

$$c_{ss} = \frac{4}{3} = 1.33 \text{ volts}$$

Steady state error $e_{ss} = \lim_{s \rightarrow 0} \frac{s \left(\frac{2}{s} \right)}{1 + \frac{2}{(s+1)(s+1)}} (1)$

$$e_{ss} = \frac{2}{3} = 0.66 \text{ volts}$$

$$\text{Settling time } t_s = \frac{4}{\delta\omega_n} = \frac{4}{(0.577)\sqrt{3}} = 4\text{msec}$$

$$\text{Peak time } t_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} = \frac{3.14}{\sqrt{3} * \sqrt{1-0.577^2}} = 2.22\text{msec}$$

$$\begin{aligned} \text{Peak overshoot } \% M_p &= e^{-\pi\delta/\sqrt{1-\delta^2}} \times 100 \\ &= e^{-\pi(0.577)/\sqrt{1-0.577^2}} \times 100 = 10.8\% \end{aligned}$$

Case (ii) Type-0 system with gain K = 1

$$G(s) = \frac{1}{(\tau s + 1)(\tau s + 1)} \text{ where } \tau = 1\text{sec}$$

The closed loop transfer function for this system is,

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{(s+1)(s+1)}}{1 + \frac{1}{(s+1)(s+1)}} = \frac{1}{s^2 + 2s + 2}$$

Rearranging the above equation to a general second order form, then

$$\frac{C(s)}{R(s)} = \frac{1}{2} \left(\frac{2}{s^2 + 2s + 2} \right)$$

The general second order closed loop transfer function for type-0 system,

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Comparing the above two equations, the following parameters are obtained.

Natural frequency $\omega_n^2 = 2$;

$$\begin{aligned} \omega_n &= \sqrt{2} \\ &= 1.414 \text{ rad/sec} \end{aligned}$$

$$2\delta\omega_n = 2;$$

$$\begin{aligned} \text{Damping factor } \delta &= \frac{1}{\omega_n} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$= 0.707$$

Steady state value of the given system for step input is given by

$$C_{ss} = \lim_{s \rightarrow 0} sG(s)$$

We know that $\frac{C(s)}{R(s)} = \frac{1}{2} \left(\frac{2}{s^2 + 2s + 2} \right)$

For a step input of 2volts, $R(s) = \frac{2}{s}$.

$$C(s) = \frac{2}{s} \left(\frac{1}{2} \right) \left(\frac{2}{s^2 + 2s + 2} \right)$$

Applying final value theorem,

$$C_{ss} = \lim_{s \rightarrow 0} \left(s \frac{2}{s} \left(\frac{1}{2} \right) \left(\frac{2}{s^2 + 2s + 2} \right) \right)$$

$$C_{ss} = \frac{2}{2} = 1 \text{ volts.}$$

Steady state error $e_{ss} = \lim_{s \rightarrow 0} \frac{s \left(\frac{2}{s} \right)}{1 + \frac{1}{(s+1)(s+1)}} \quad (1)$

$$e_{ss} = \frac{2}{2} = 1 \text{ volts.}$$

Settling time $t_s = \frac{4}{\delta \omega_n} = \frac{4}{(0.707)\sqrt{2}} = 4 \text{ msec.}$

Peak time $t_p = \frac{\pi}{\omega_n \sqrt{1-\delta^2}} = \frac{3.14}{\sqrt{2} * \sqrt{1-0.707^2}} = 3.14 \text{ msec.}$

Peak overshoot % $M_p = e^{-\pi\delta/\sqrt{1-\delta^2}} \times 100$
 $= e^{-\pi(0.707)/\sqrt{1-0.707^2}} \times 100 = 4.325\%$

Case (iii) Type-1 system with gain K = 2

$$G(s) = \frac{2}{s(\tau s + 1)} \quad \text{where } \tau = 1 \text{ sec}$$

The closed loop transfer function for this system is,

$$\frac{C(s)}{R(s)} = \frac{\frac{2}{s(\tau s + 1)}}{1 + \frac{2}{s(\tau s + 1)}} = \frac{2}{s^2 + s + 2}$$

The general second order closed loop transfer function for type-0 system,

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Comparing the above two equations, the following parameters are obtained.

Natural frequency $\omega_n^2 = 2$;

$$\begin{aligned}\omega_n &= \sqrt{2} \\ &= 1.414 \text{ rad/sec}\end{aligned}$$

$$2\delta\omega_n = 1;$$

Damping factor $\delta = \frac{0.5}{\omega_n}$

$$\begin{aligned}&= \frac{0.5}{\sqrt{2}} \\ &= 0.3536\end{aligned}$$

Steady state value of the given system for step input is given by

$$c_{ss} = \lim_{s \rightarrow 0} sG(s)$$

We know that $\frac{C(s)}{R(s)} = \left(\frac{2}{s^2 + s + 2} \right)$

For a step input of 2volts, $R(s) = \frac{2}{s}$.

$$C(s) = \frac{2}{s} \left(\frac{2}{s^2 + s + 2} \right)$$

Applying final value theorem,

$$c_{ss} = \lim_{s \rightarrow 0} \left(s \frac{2}{s(s^2 + s + 2)} \right)$$

$$c_{ss} = \frac{4}{2} = 2 \text{ volts.}$$

$$\text{Steady state error } e_{ss} = \lim_{s \rightarrow 0} \frac{s \left(\frac{2}{s} \right)}{1 + \frac{2}{s(\tau s + 1)}} \quad (1)$$

$$e_{ss} = 0 \text{ volts.}$$

$$\text{Settling time } t_s = \frac{4}{\delta \omega_n} = \frac{4}{(0.3536)\sqrt{2}} = 8 \text{ msec.}$$

$$\text{Peak time } t_p = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}} = \frac{3.14}{\sqrt{2} * \sqrt{1 - 0.3536^2}} = 3 \text{ msec.}$$

$$\begin{aligned} \text{Peak overshoot } \% M_p &= e^{-\pi \delta / \sqrt{1 - \delta^2}} \times 100 \\ &= e^{-\pi(0.3536) / \sqrt{1 - 0.3536^2}} \times 100 = 30.5\% \end{aligned}$$

Case (iv) Type-1 system with gain K = 1

$$G(s) = \frac{1}{s(\tau s + 1)} \quad \text{where } \tau = 1 \text{ sec}$$

The closed loop transfer function for this system is,

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s(\tau s + 1)}}{1 + \frac{1}{s(\tau s + 1)}} = \frac{1}{s^2 + s + 1}$$

Rearranging the above equation to a general second order form, then

$$\frac{C(s)}{R(s)} = \left(\frac{1}{s^2 + s + 1} \right)$$

The general second order closed loop transfer function for type-0 system,

$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Comparing the above two equations, the following parameters are obtained.

Natural frequency $\omega_n^2 = 1$;

$$\begin{aligned}\omega_n &= \sqrt{1} \\ &= 1 \text{ rad/sec}\end{aligned}$$

$$2\delta\omega_n = 1;$$

Damping factor $\delta = \frac{0.5}{\omega_n}$

$$\begin{aligned}&= \frac{0.5}{1} \\ &= 0.5\end{aligned}$$

Steady state value of the given system for step input is given by

$$c_{ss} = \lim_{s \rightarrow 0} sG(s)$$

We know that $\frac{C(s)}{R(s)} = \left(\frac{1}{s^2 + s + 1} \right)$

For a step input of 2volts,

$$C(s) = \frac{2}{s} \left(\frac{1}{s^2 + s + 1} \right)$$

Applying final value theorem,

$$c_{ss} = \lim_{s \rightarrow 0} \left(s \frac{2}{s} \left(\frac{1}{s^2 + s + 1} \right) \right)$$

$$c_{ss} = \frac{2}{1} = 2 \text{ volts.}$$

$$\text{Steady state error } e_{ss} = \lim_{s \rightarrow 0} \frac{s \left(\frac{2}{s} \right)}{1 + \frac{1}{s(\tau s + 1)}} (1)$$

$$e_{ss} = 0 \text{ volts.}$$

$$\text{Settling time } t_s = \frac{4}{\delta \omega_n} = \frac{4}{(0.5)(1)} = 8 \text{ msec.}$$

$$\text{Peak time } t_p = \frac{\pi}{\omega_n \sqrt{1 - \delta^2}} = \frac{3.14}{1 * \sqrt{1 - 0.5^2}} = 3.62 \text{ msec.}$$

$$\begin{aligned} \text{Peak overshoot } \% M_p &= e^{-\pi \delta / \sqrt{1 - \delta^2}} \times 100 \\ &= e^{-\pi(0.5) / \sqrt{1 - 0.5^2}} \times 100 = 16.36\% \end{aligned}$$

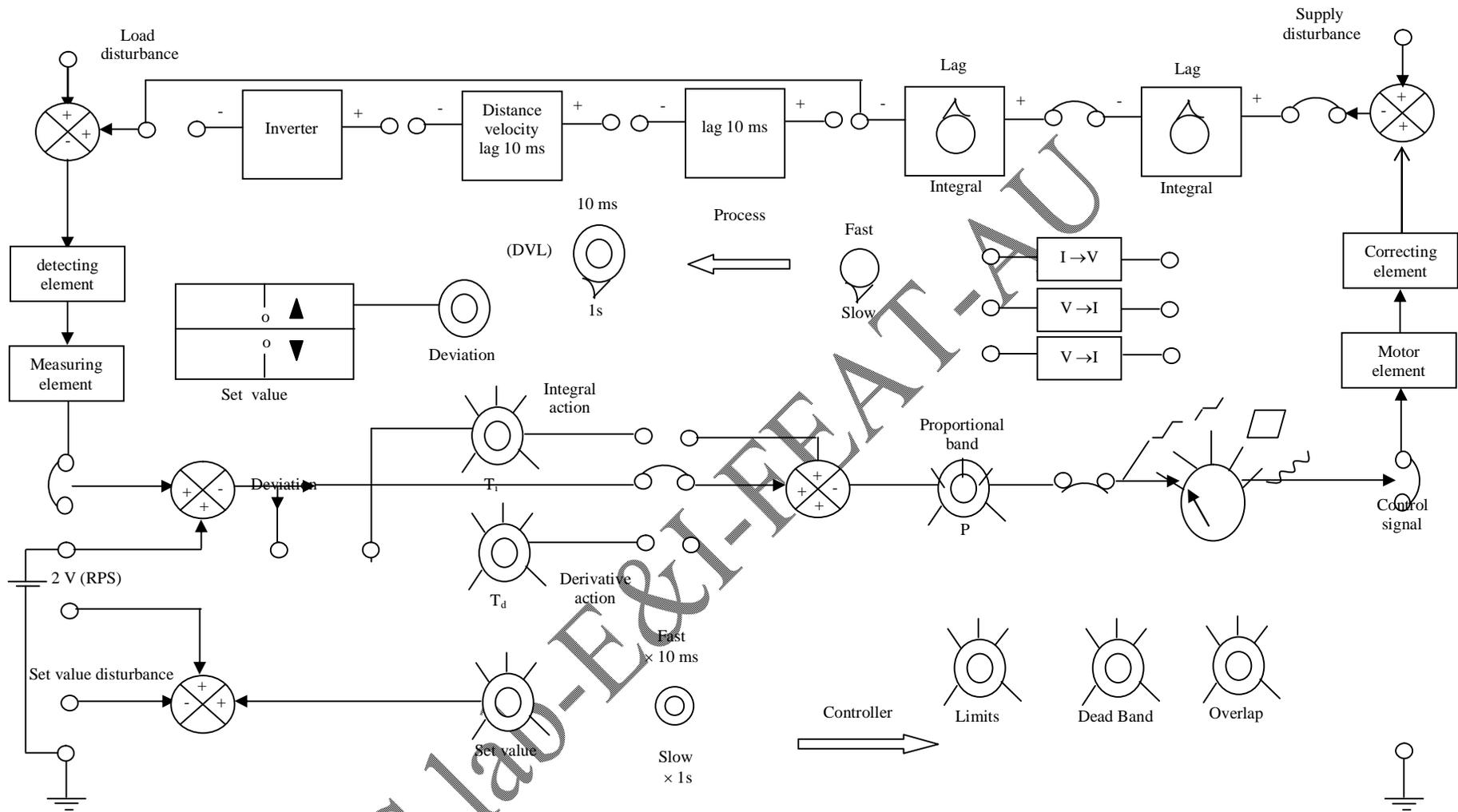


Fig. 2 Closed loop connection diagram for a second order type - 0 system

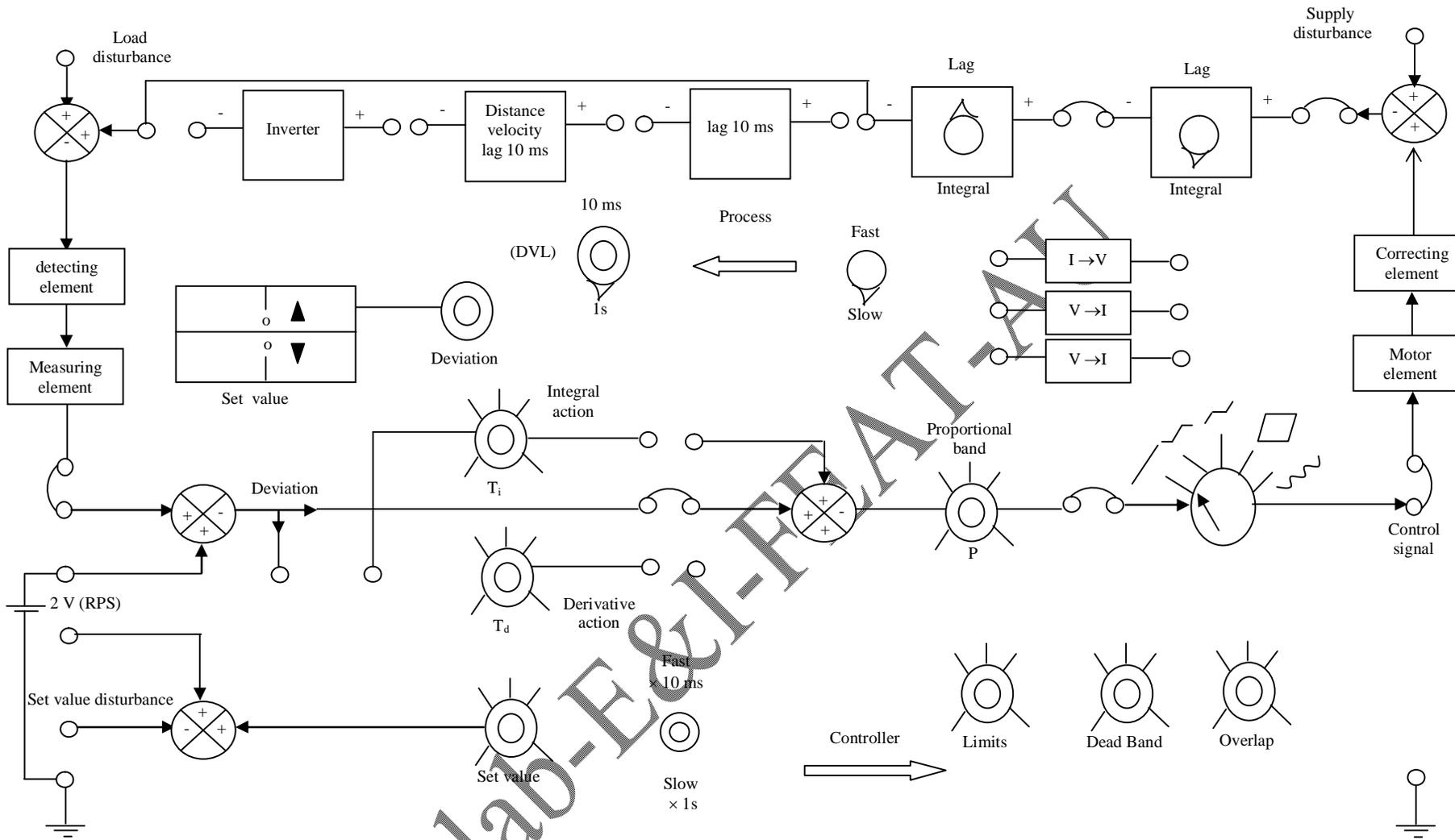


Fig. 3 Closed loop connection diagram for a second order type-1 system

PROCEDURE

For Type-0 system

10. Set the given type - 0 process in Process Control Simulator by choosing two lag terms and time constant of 1sec using appropriate switch positions as shown in Fig. 2.
11. Set the gain K using % PB Knob in PCS where $K = \left[\frac{100}{\% PB} \right]$.
12. Make the connections as shown in Fig. 2 to enable a closed loop configuration.
13. Apply a step or pulse input of 2V using the DC voltage source (RPS or Signal generator).
14. Trace the output response displayed in CRO and calculate the transient and steady state parameters as explained in the theoretical calculation.
15. Compare the obtained practical values with the theoretical calculated values.

For Type-1 system

1. Set the given type-1 process in Process Control Simulator by choosing one lag term and one integrator with 1 sec time constant.
2. Set the gain K using % PB Knob in PCS where $K = \left[\frac{100}{\% PB} \right]$
3. Make the connections as shown in Fig. 3 to enable a closed loop configuration.
4. Apply a step or pulse input of 2V.
5. Trace the output response displayed in CRO and calculate the transient and steady state parameters as explained in the theoretical calculation.
6. Compare the obtained practical values with the theoretical calculated values.

Table 1 Time response of a Second order -Type-0 and Type-1 System

Step input $r(t) = 2u(t)$ or (2 volts)

SI. No.	G(s) with $\tau = 1$ sec	Closed loop transfer function $\frac{C(s)}{R(s)}$	δ	ω_n	Steady state value $c(t)$ (v)		Steady state error e_{ss} (v)		Settling time (t_s) ms		Peak time (t_p) ms		% overshoot (%Mp)	
					Theo.	Prac.	Theo.	Prac.	Theo.	Prac.	Theo.	Prac.	Theo.	Prac.
Type 0 K=2	$\frac{2}{(\tau s + 1)(\tau s + 1)}$	$\frac{2}{3} \left(\frac{3}{s^2 + 2s + 3} \right)$	0.577	$\sqrt{3}$	1.33		0.66		4		2.22		10.3	
Type 0 K=1	$\frac{1}{(\tau s + 1)(\tau s + 1)}$	$\frac{1}{2} \left(\frac{2}{s^2 + 2s + 2} \right)$	0.707	$\sqrt{2}$	1		1		4		3.14		4.32	
Type 1 K=2	$\frac{2}{s(\tau s + 1)}$	$\left(\frac{2}{s^2 + s + 2} \right)$	0.3536	$\sqrt{2}$	2		0		8		3		30.5	
Type 1 K=1	$\frac{1}{s(\tau s + 1)}$	$\left(\frac{1}{s^2 + s + 1} \right)$	0.5	1	2		0		8		3.62		16.36	

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RESULT

Hence the time response analysis of type-0 and type-1 second order systems has been analyzed using process control simulator. The transient response parameters obtained practically were compared with the theoretical values.